CS 5860

Tue Oct 25, 2011

Induction continued

We use the induction form for all computation on numbers.

For example, to decide whether a number is zero, we compute with $ind(x; *; u,i.\_\_)$ where $\_\_$ is any computation form that "aborts" like $ap(0;0)$.

We can use ind to prove statements such as

1. $\forall x.(Z(x) \vee \sim Z(x))$

2. $\forall x,y.(Eq(x,y) \vee \sim Eq(x,y))$

3. $\forall x (\sim Z(x) \Rightarrow \exists y. Suc(y,x))$

We can use induction to prove

4. $\forall x,y. \exists z. Add(x,y,z)$

5. $\forall x,y. \exists z. Mult(x,y,z)$

We will prove 4 below. You should try 1, 2, 3, 5 as exercises. Also try 6 below.

6. Define $x<y$ iff $\exists z.(x+z=y \,\&\, z \neq 0)$. Show:

   (a) $x < s(x)$ (b) $(x<y \,\&\, y<z) \Rightarrow x<z$ (c) $\sim(x<x)$