1. Show that a choice function $C$ satisfies axioms $\alpha$ and $\beta$ iff it satisfies WARP (the Weak Axiom of Revealed Preference). (See the class notes for the relevant definitions.)

2. Suppose $X = \{x, y, z\}$. Consider a choice function $C : P(X) \rightarrow P(X)$ such that $C(\{x, y\}) = \{x\}$, $C(\{x, z\}) = \{z\}$ and $C(\{y, z\}) = \{y\}$. Does this choice function satisfy Sen’s $\alpha$? Does it satisfy Sen’s $\beta$?

3. Let $\succ$ be a binary relation on a finite set $X$. Define $\succeq$ by: $x \succeq y$ if and only if $y \not\succ x$. Show

   (a) If $\succeq$ is complete then $\succ$ is asymmetric.

   (b) If $\succeq$ is transitive then $\succ$ is negatively transitive.

4. Suppose that $\succ$ is transitive (but not necessarily negatively transitive), and define $c(\cdot, \succ)$ as in class. Show that Sen’s axiom $\alpha$ holds, but show by example that Sen’s $\beta$ may fail to hold.

5. A binary relation that is reflexive, symmetric, and transitive is called an equivalence relation. Suppose that $\succ$ is a strict preference relation on a finite set $X$. Then by Proposition 2.4 of Kreps we know that $\sim$ is an equivalence relation on $X$. For each $x \in X$, define $I(x) = \{y \in X : y \sim x\}$; $I(x)$ is called the equivalence class of $x$. Show:

   (a) The sets $I(x)$ partition $X$. (A collection of sets $\{A_1, \ldots, A_N\}$ partitions $X$ if each $x \in X$ is in at least one $A_i$ and $A_i \cap A_j = \emptyset$ for all $i \neq j$.)
(b) The sets $I(x)$ are strictly ranked. (The equivalence classes are strictly ranked if, for all $x, y \in X$: (1) if $I(x) \neq I(y)$, then either $x \succ y$ or $y \succ x$, and (2) if $x \succ y$ then $x' \succ y'$ for all $x' \in I(x)$ and $y' \in I(y)$.)

6. **GRAD:** In the statement of Sen’s $\alpha$ and $\beta$ we allow the sets $A$ and $B$ to be any subsets of $X$. So when we proved that these axioms imply that the revealed preference relation is asymmetric and negatively transitive we allowed ourselves to use information about choices from arbitrary subsets of $X$. We want to know whether there is a smaller class of subsets of $X$ such that the claim in the revealed preference theorem is true if $\alpha$ and $\beta$ are satisfied on this smaller class of sets. Suppose that the cardinality of $X$ is $N$ and for each integer $n \leq N$ let $S_n$ be the collection of all non-empty subsets of $X$ of cardinality less than or equal to $n$. Find the smallest $n > 1$ such that the following claim is true: If a choice function satisfies Sen’s $\alpha$ and $\beta$ on $S_n$ then there is a preference order $\succ$ defined on $X$ such that $c(A, \succ) = c(A)$ for all $A \in S_n$.

7. **GRAD:** In class in the proof of the revealed preference theorem we defined strict revealed preference. Weak revealed preference is defined as follows: $x \succeq y$ if $x \in C(\{x,y\})$. Define induced strict revealed preference $\succ^*$ from revealed preference $\succeq$ by: $x \succ^* y$ if $x \succeq y$ and $y \not\succeq x$. Are strict revealed preference and induced strict revealed preference the same relation?