Decision Theory Prelim

March 11, 2014

There are 4 questions; question 4(b) if for grad students only (i.e., those of you taking CS 5846 or ECON 6760). Please answer each question in a separate blue book. The test is out of 35 (40 if you are taking CS 5846 or ECON 6760); you have 75 minutes. Good luck!

1. [16 points] A number in $\{2, 3, 4, 5\}$ is written on a piece of paper which is turned over (so that you don't see it). You also have to write a number in $\{2, 3, 4, 5\}$ on another piece of paper (without seeing what has already been written). If you match the number written, you get what you wrote; if your number is less than the number written, you get two more than what you wrote; if your number is more than the number written, you get two less than what is written. For example, if you write 3, and the number already written is 3, then you get 3; if the number already written is 4, you get 5 (two more than what you wrote, since what you wrote is less than 4); if the number written is 2, then you get 0 (two less than what is written).

   (a) [3 points] Express your problem as a decision problem, by describing carefully the set of states, acts, and outcomes; also write down the payoff matrix.

   (b) [2 points] What is the best act according to the maximin rule? (If there is more than one best act in this part and all the remaining parts, give them all. You also need to explain why it is best. It's not enough to just write "x is the best".)

   (c) [4 points] Show that, for all $\alpha \in [0, 1]$, neither 3 nor 5 are best with respect to opt^α (the optimism-pessimism rule with parameter α).

   (d) [3 points] What is the best act according to the minimax regret rule?

   (e) [4 points] If all the outcomes were squared (e.g., if you both played 3, you would both get 9), would the answer to (b) change? How about the answer to (d)? (Again, it's not enough to just say "yes" or "no". You need to explain in one sentence whether things change, and if they do, what the change is.)

2. [6 points] Define the lexicographic order $>$ on pairs of numbers (a, b) by taking $(a, b) > (a', b')$ if either (i) $a > a'$ or (ii) $a = a'$ and $b > b'$. Show that the lexicographic order is in fact a preference relation.

3. [8 points] Let $X = \{a, b, c\}$ and suppose we have a choice function c such that $c(\{a, c\}) = \{c\}$, $c(\{a, b\}) = \{a, b\}$, and $c(\{b, c\}) = \{b, c\}$.

   (a) Is there a preference relation $\succ$ such that $C(\cdot) = C(\cdot, \succ)$?

   (b) Is there a strict partial order $\succ$ such that $C(\cdot) = C(\cdot, \succ)$?

   Explain why or why not.

4. Suppose that a DM is an expected utility maximizer with payoff function $u(x) = \ln x$. The individual has a certain wealth $w > 0$. There are two financial assets, a and b. With probability p such that $0 < p < 1$, a pays off 2 dollars gross per dollar invested (that is, it pays back the original investment plus one additional dollar per dollar invested) and b pays off 0. With probability $1 - p$, a pays off 0 and b pays off 2 dollars gross per dollar invested.

   (a) [5 points] How should the individual divide his wealth between assets a and b? Hint: What *share* of his wealth should go to a, and what *share* to b? (Note that we are assuming that the individual will be fully invested, so all of his money will go to either a or b.)

   (b) [GRAD] [5 points each for (i) and (ii)] Now suppose that $u(x) = x^\gamma/\gamma$ for $\gamma < 1$, $\gamma \neq 0$. (Recall that $u(x) = x^\gamma/\gamma$ and $u(x) = \ln(x)$ are the two types of CRRA utility functions.) Find the optimal distribution of shares.

      (i) What happens to the optimal distribution as the coefficient of relative risk aversion ($1 - \gamma$ in the equation above) converges to ∞?

      (ii) What happens to the optimal distribution as the coefficient of relative risk aversion converges to 0?

      [Hint: you don't have to completely solve for the general expression for the optimal distribution to figure out what happens as the coefficient of relative risk aversion converges to $0/\infty$.]