Computer Science 576: HW2

Handed out: Sept. 23, 2010. Due: Oct. 7, 2010

1. (a) Suppose that you are an insurance broker, trying to decide whether to offer fire insurance to a business located in an old wooden structure with an oil furnace in the basement. What is a reasonable state space and outcome space for the decision? How do things change if you have reason to believe that the owner of the business was a convicted arsonist? (There's no "right" or "wrong" answer here. But you should argue for the reasonableness of your choices.)

   (b) Recall the "better red than dead" example from Prof. Halpern's notes. Define a reasonable state space that is independent of the acts "arm and "disarm". (The outcome space is still "dead", "red", "status quo", and "improved society".) Again, there is no unique "right" answer. But whatever answer you give, you should argue that the acts are independent of the states you choose.

2. Prove the following proposition from Prof. Halpern's notes:
   Proposition: Let f be an increasing function. Then $\text{maximin}(u) = \text{maximin}(f(u))$ and $\text{maximax}(u) = \text{maximax}(f(u))$. Show by example that $opt^{\alpha}(u)$ may not be the same as $opt^{\alpha}(f(u))$, and that $regret(u)$ may not be the same as $regret(f(u))$.

3. Prove the following proposition from Professor Halpern's notes:
   Proposition: Let f be a positive affine transformation. Then

   - $\text{maximin}(u) = \text{maximin}(f(u))$
   - $\text{maximax}(u) = \text{maximax}(f(u))$
   - $opt^{\alpha}(u) = opt^{\alpha}(f(u))$
   - $regret(u) = regret(f(u))$.

4. Recall the algorithm for the location problem: The robot goes to $+1$, then -2, then $+4$, then -8, $\ldots$, until it finds the object.

   (a) Show that this algorithm has a competitive ratio of 9. That is, show that compared that, no matter where the object is located, it will take the robot at most 9 times as many steps to find the object using this algorithm as it would to go directly to the object.

(b) Show that if $k < 9$, then for all c, there is a location N_c for the object such that the robot will take more than $kN_c + c$ steps to find the object.

5. GRADUATE: Show that no algorithm for the robot location problem has competitive ratio better than 3. (Hint: first show that, without loss of generality, there exist two sequences $N_1, N_2, \ldots$ and $N'_1, N'_2, \ldots$ such that $0 < N_1 < N_2 < \ldots$ and $0 < N'_1 < N'_2 < \ldots$ such that the robot goes to $+N_1, -N'_1, +N_2, -N'_2, \ldots$, until it finds the object.)

6. Show that $a \geq^3_{\mathcal{P}} a'$ implies $a \geq^4_{\mathcal{P}} a'$, where $\mathcal{P}$ is a set of probability distributions and $\geq^3_{\mathcal{P}}$ and $a^4_{\mathcal{P}}$ are the partial orders defined in Prof. Halpern's notes.