Professors Blume, Halpern and Easley — Econ 476/676
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Final Exam

INSTRUCTIONS, SCORING: The points for each question are specified at the beginning of the question. There are 160 points on the exam. Please show your work, so that we can give you partial credit. Good luck!

1. (20 points) Consider the binary relation $\succ$ defined as follows on the reals: For $x, y \in \mathbf{R}$, $x \succ y$ if $x - y$ is an integer.

   (a) Is this binary relation transitive?

   (b) Is this relation negatively transitive?

2. (20 points) Let Ω be a finite state space, O a set of outcomes, and $u : O \to \mathbf{R}$ a payoff function. Let D denote a finite set of acts $d : \Omega \to O$. Define $u_d(\omega) = u\big(d(\omega)\big)$. Denote by $r_d(\omega)$ the regret from using d in state w.

   (a) Define $r_d(\omega)$.

   (b) Let p be a probability distribution on Ω. Define $d \succeq_1 e$ iff $E_p[u_d(\omega)] \geq E_p[u_e(\omega)]$. Define $d \succ_2 e$ if and only if $E_p[r_d(\omega)] \leq E_p[r_e(\omega)]$. Show that $\succeq_1 = \succeq_2$.

   (c) Let $d \succeq_3 e$ if and only if $\min_\omega u_d(\omega) \geq \min_\omega u_e(\omega)$. Let $d \succeq_4 e$ if and only if $\max_\omega r_d(\omega) \leq \max_\omega r_e(\omega)$. Show that if $\max_d u_d(\omega)$ is a constant independent of ω, then $\succeq_3 = \succeq_4$.

3. (20 points) This problem computes some minimax regret rules. In this problem the state space is $\Omega = [0, 1]$, the action set is $D = \{0, 1\}$ and the payoff is $u_d(\omega) = -(d - \omega)^2$.

   (a) Compute the regret of each act and identify which decisions are optimal under the minimax regret criterion.

(b) Now suppose that randomized decision procedures can be used. The set of rules is now $p \in [0, 1]$, where p is the probability of choosing $d = 0$. The payoff to rule d in state ω is its expected payoff, $U(\omega, p) = -p\omega^2 - (1 - p)(1 - \omega)^2$. Find the randomized rules which are optimal under the minimax regret criterion.

4. (20 points) Given two acts a and b, say that a *weakly dominates* b (with respect to a utility function u) if for all states $s \in S$, we have $u(a(s)) \geq u(b(s))$ and for some $s' \in S$, we have $u(a(s)) > u(b(s))$. Suppose that a weakly dominates b. Which of $a \succ b$, $a \sim b$, and $b \succ a$ can/must happen with each of the following decision rules:

   (a) maximin,

   (b) minimax regret,

   (c) the principle of insufficient reason.

   If you think one of these can't happen, given an example that demonstrates it; if you think it must happen, explain why. (Note: 1-2 sentence explanations suffice in each case.)

5. (20 points) Consider the following Bayesian network containing 3 Boolean random variables (that is, the random variables have two truth values—*true* and *false*):

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   A
  / \
 B   C
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   Suppose the Bayesian network has the following conditional probability tables (where X and $\overline{X}$ are abbreviations for $X = true$ and $X = false$, respectively):

$$\begin{array}{l} \Pr(A) = .1 \\ \Pr(B \mid A) = .7 \\ \Pr(B \mid \overline{A}) = .2) \\ \Pr(C \mid A) = .4) \\ \Pr(C \mid \overline{A}) = .6) \end{array}$$

   (a) What is $\Pr(\overline{B} \cap C \mid A)$?

(b) What is $\Pr(A \mid \overline{B} \cap C)$?

(c) Suppose we add a fourth node labeled D to the network, with edges from both B and C to D. For the new network

 (i) Is A conditionally independent of D given B?

 (ii) Is B conditionally independent of C given A?

 In both cases, explain your answer.

6. (20 points) Consider the following CP (ceteris paribus) network:

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A --> B --> C
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with the following conditional preference tables:

$a \succ \overline{a}$

a	$b \succ \overline{b}$
$\overline{a}$	$\overline{b} \succ b$

b	$c \succ \overline{c}$
$\overline{b}$	$\overline{c} \succ c$

(a) Does it follow from this CPnet that $abc \succ a\overline{b}\overline{c}$? (Explain why or why not; a simple yes or no will not get any points.)

(b) Does it follow from this CPnet that $\overline{a}\overline{b}\overline{c} \succ a\overline{b}c$? (Again, explain why or why not.)

7. (20 points) Consider the lotteries:

A If Hillary Clinton is the next President then you win \$1 with probability p and \$0 with probability $1 - p$; if she is not then you win \$1 with probability 0 and \$0 with probability 1.

B If Hillary Clinton is the next President then you win \$1 with probability 0 and \$0 with probability 1; if she is not then you win \$1 with probability q and \$0 with probability $1 - q$.

C You win \$1 with probability 1/2 and \$0 with probability 1/2.

Suppose that $\$1 \succ \$0$.

(a) Suppose p and q are such that the decision maker is indifferent between lotteries A and B. Assuming that this decision maker is a subjective expected utility maximizer (with a state independent payoff function) determine the decision maker's subjective probability that Hillary Clinton is the next President.

(b) Now suppose that $p = q = 1$, the decision maker may or may not be indifferent between A and B and for these values of p and q we have $C \succ A$ and $C \succ B$. Show that in this case the decision maker cannot be a subjective expected utility maximizer.

(c) For the values of p and q and preferences in part (b) suppose that the decision maker acts as if he has a utility function on money u, a set of probabilities Π on the states, Hillary and not-Hillary, and evaluates each lottery using the element of Π that minimizes the expected utility of the lottery. What do you know about Π?

8. (20 points) A decision maker has to allocate his initial wealth $w_0 > 0$ between two assets. One asset is risk free and pays a certain return of 1 for every dollar invested. The other is risky; its return per dollar invested is given by the random variable r which takes on values $\bar{r}$ or $\underline{r}$. Let $\alpha \in [0, 1]$ be the fraction of wealth invested in the risky asset. So future wealth is $w = (1 - \alpha)w_0 + \alpha w_0 r$.

   (a) In this part of the question the decision maker is a von-Neumann Morgenstern expected utility maximizer with payoff function for future wealth $u(w)$. Assume that $u'(w) > 0$ and $u''(w) < 0$ for all w and that the expectation of r is greater than one. Show that the optimal choice of α is greater than 0.

   (b) Now the distribution of r is uncertain. It depends on the state of nature which is either H or L. In state H, $r = \bar{r}$ with probability p and $r = \underline{r}$ with probability $1 - p$. In state L, $r = \bar{r}$ with probability q and $r = \underline{r}$ with probability $1 - q$. Suppose that for each α the decision maker's preferences over lotteries satisfy the Anscombe and Aumann axioms. Let π represent the decision maker's subjective probability of state H and let $u(w)$ be his payoff function for future wealth. Write the decision maker's expected utility as a function of α.

(c) Continuing with the setup in part (b) suppose that $1 > \pi > 0$. Show that if we allow the payoff function to depend on the state, i.e. $U(w, H) \neq U(w, L)$, then the subjective probability of state H can be represented by any $\hat{\pi}$ with $1 > \hat{\pi} > 0$.