Allais Paradox

The set of prizes is $X = \{\$0, \$1,000,000, \$5,000,000\}$.

- Which probability do you prefer:
  $p_1 = (0.00, 1.00, 0.00)$ or $p_2 = (0.01, 0.89, 0.10)$?
- Which probability do you prefer:
  $p_3 = (0.90, 0.00, 0.10)$ or $p_4 = (0.89, 0.11, 0.00)$?

Many subjects report: $p_1 \succ p_2$ and $p_3 \succ p_4$

Inconsistent with EUT

Suppose (u_0, u_1, u_5) represents $\succ$.

Then $p_1 \succ p_2$ implies

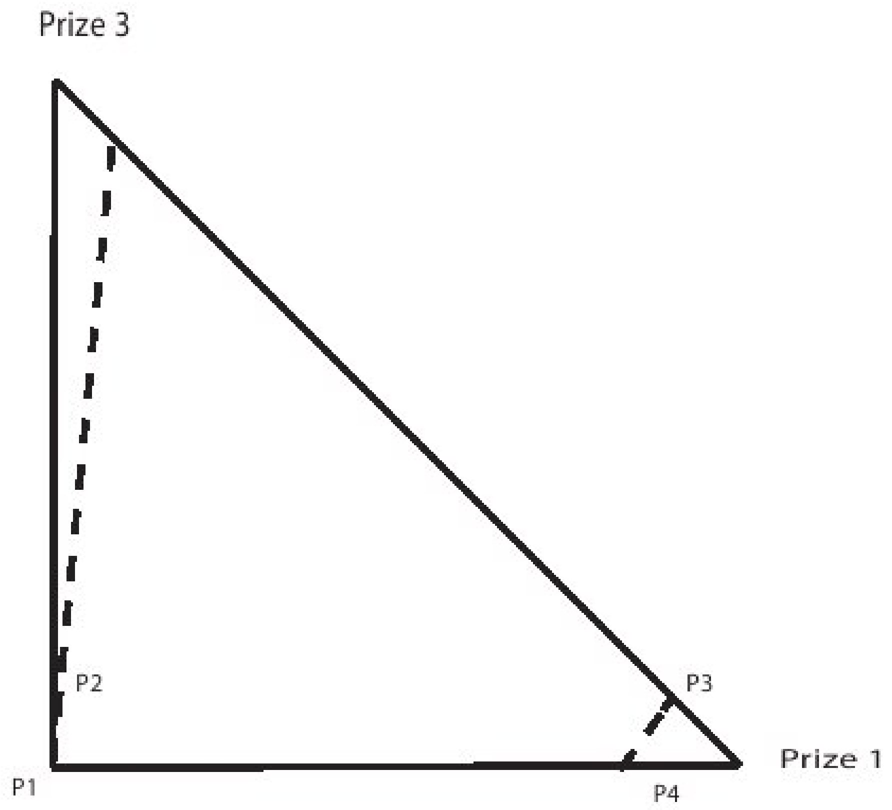
$$u_1 > .01u_0 + .89u_1 + .1u_5$$
$$.11u_1 - .01u_0 > .1u_5$$
$$.11u_1 + .89u_0 > .1u_5 + .9u_0.$$

So $p_4 \succ p_3$.

What axiom is violated?

Independence

Inconsistent with Parallel Linear Indifference Curves



Ellsberg Paradox

There is one urn with with 300 balls: 100 of these balls are red (R) and the rest are either blue (B) or yellow(Y). Consider the following two choice situations:

I: a. Win \$100 if a ball drawn from the urn is R and nothing otherwise.

a'. Win \$100 if a ball drawn from the urn is B and nothing otherwise.

II: b. Win \$100 if a ball drawn from the urn is R or Y and nothing otherwise.

b'. Win \$100 if a ball drawn from the urn is B or Y and nothing otherwise.

Inconsistent with SEU

Suppose a decision maker's preferences are such that $a \succ a'$ and $b' \succ b$.

If there are subjective probabilities then the first choice implies that the probability of a red ball is greater than the probability of a blue ball and the second choice implies the reverse.

Which axiom is violated?

Violation of Savage's Independence Axiom

State space, $S = \{R, B, Y\}$

Set of prizes, $X = \{0, 100\}$

- Lottery a is $a : S \to X$ such that $a(R) = 100, a(B) = 0, a(Y) = 0$.
- Lottery a$\prime$ is $a\prime : S \to X$ such that $a\prime(R) = 0, a\prime(B) = 100, a\prime(Y) = 0$.
- Lottery b is $b : S \to X$ such that $b(R) = 100, b(B) = 0, b(Y) = 100$.
- Lottery b$\prime$ is $b\prime : S \to X$ such that $b\prime(R) = 0, b\prime(B) = 100, b\prime(Y) = 100$.

Let $E = \{R, B\}$ and note that $S = E \cup \{Y\}$. On E, $a = b$ and $a\prime = b\prime$. Further $a(Y) = a\prime(Y)$ and $b(Y) = b\prime(Y)$. We have $a \succ a\prime$. The independence axiom then implies that $b \succ b\prime$. But we have $b\prime \succ b$. So the independence axiom is violated.

Multiple Priors

Suppose that the decision maker's uncertainty can be represented by a set probabilities for blue and yellow and he chooses using the most pessimistic belief.

Could this decision maker chose the observed outcomes in the Ellsberg Paradox?

Let $p = (1/3, p_B, p_Y)$ be a probability on the draw. The decision maker has a set P of probabilities.

In any choice situation the decision maker chooses using a maximin rule: For each lottery evaluate expected utility using the probability in P that minimizes expected utility. Select the lottery that maximizes over these minimized values. (See Professor Halpern's lecture on decision rules.)

- Note that $a \succ a\prime$ implies that

$$1/3u(100){+}2/3u(0) > \min_{p_B}\{p_B u(100){+}(1{-}p_B)u(0)\}$$

- Similarly $b\prime \succ b$ implies that

$$1/3u(0){+}2/3u(100) > \min_{p_B}\{(1{-}p_B)u(100){+}p_B u(0)\}$$

- Let $\underline{p_B}$ be the minimum $p_B \in P$ and $\overline{p_B}$ be the maximum $p_B \in P$.
- The first equation above implies that $1/3 > \underline{p_B}$.
- The second equation above implies that $\overline{p_B} > 1/3$.
- So P must contain some $p_B < 1/3$ and some $p_B > 1/3$.

Maximin Expected Utility

Let $\mathcal{P}$ be a set of probabilities on the prizes X.

Professor Halpern defined Maximin Expected Utility of the act a as

$$\underline{\mathrm{E}}P(u_a) = \inf_{Pr \in \mathcal{P}} \{E_{Pr}(u_a) : Pr \in \mathcal{P}\}$$

Then for a decision maker using the Maximin Expected Utility Decision Rule we have

$$a \succ b \text{ if and only if } \underline{\mathrm{E}}\mathrm{P}(\mathrm{u_a}) > \underline{\mathrm{E}}\mathrm{P}(\mathrm{u_b})$$

Gilboa and Schmeidler, *Journal of Mathematical Economics*, 1989 provide an axiomatic foundation for this decision rule.

Maximin Violates the Independence Axiom

Let $S = \{s_1, s_2\}$ and $\mathcal{P} = \{(1/4, 3/4), (2/3, 1/3)\}$.

Consider acts $a = (1, 1), b = (2, 0), c = (0, 2)$ where the first component is the prize on state 1 and so on. Suppose that $u(x) = x$ for prizes x.

Then $\underline{\mathrm{E}}P(u_a) = 1 > \underline{\mathrm{E}}P(u_b) = 1/2$. So $a \succ b$.

Now $1/2a + 1/2c = (1/2, 3/2)$ and $1/2b + 1/2c = (1, 1)$.

So $\underline{\mathrm{E}}P(u_{1/2a+1/2c}) = 5/6 < \underline{\mathrm{E}}P(u_{1/2b+1/2c}) = 1$.

So $1/2b + 1/2c \succ 1/2a + 1/2c$.

Gilboa and Schmeidler replace indpendence with a weaker axiom.

A cab was involved in a hit and run accident last night. Two cab companies, Green and Blue, operate in the city.

You know:

- 85% of the cabs in the city are Green the rest are Blue.
- A witness identified the cab as Blue.
- Tests have shown that in similar cirumstances witnesses correctly identify each of the two cabs 80% of the time and misidentify them 20% of the time.

What is the probability that the cab involved in the acciden was Blue?

The correct answer is

$$\begin{aligned} Pr(B|idB) &= \frac{Pr(idB|B)Pr(B)}{Pr(idB)} \\ &= \frac{(.8)(.15)}{(.8)(.15)+(.2)(.85)} \\ &= .41 \end{aligned}$$

Framing Effects—Kahneman and Tversky

A disease is expected to kill 600 people. Two alternative programs have been proposed:

- Program A: 200 people will be saved
- Program B: probability 1/3: 600 people will be saved probability 2/3: no one will be saved

Which Program Would you favor?

Framing Effects—Kahneman and Tversky

A disease is expected to kill 600 people. Two alternative programs have been proposed:

- Program C: 400 people will die
- Program D: probability 1/3: no one will die probability 2/3: 600 will die

Which Program Would you favor?

Framing Effects—Kahneman and Tversky

Kahneman and Tversky found:

- 72% chose A over B.
- 22% chose C over D.

But if 200 people will be saved out of 600 is the same to the decision-maker as 400 people will die out of 600, and so on, then A and C are identical and so are B and D.

Conjunction Fallacy or Failure of Extensionality

Tom is a rancher from Montana.

Which bet would you prefer?

- Win \$10 if Tom drives either a Ford or a Chevy, otherwise win nothing
- Win \$10 if Tom drives either a Chevy truck or Ford truck, otherwise win nothing

Kahneman and Tversky experiment:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which is more probable?

- Linda is a bank teller.
- Linda is a bank teller and is active in the feminist movement.

85% of subjects chose the second option.