Introduction to Interpreting Empirical Results and Hypothesis Testing

Mean & Variance

\[ \text{Mean}(x) = \bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \]
\[ \text{Variance}(x) = S^2 = \frac{\sum_{i=1}^{N} (\bar{x} - x_i)^2}{N} \]
\[ \text{StdDev}(x) = S = \sqrt{\text{Var}(x)} \]

Confidence Interval of Mean

\[ \text{StdErr}(\bar{x}) = \text{StdDev}(\bar{x}) = S/\sqrt{N} \]
\[ \pm 1S \approx 68\% \]
\[ \pm 2S \approx 95\% \]
\[ \pm 3S \approx 99\% \]

Confidence Interval

95%: \( \bar{X} - 1.96S < \text{true mean} < \bar{X} + 1.96S \)

Error Bars

- Typically 1 or 2 standard errors about mean
- Always specify what error bars are
- If 1 StdErr error bars do not overlap over regions of graph, typically assume results significantly different in regions
Hypothesis: Two Pops Have Same Mean

- t-test
- Given sample sizes, means, and variances, what are chances of seeing this large a difference in mean by chance?

\[
t = \frac{\bar{X}_1 - \bar{X}_2}{S_{pooled}\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}
\]

\[
S_{pooled} = \sqrt{\frac{(N_1 - 1)S_1^2 + (N_2 - 1)S_2^2}{N_1 + N_2 - 2}}
\]

Hypothesis Testing continued (t-test)

- calculate t statistic (see previous slide)
- Find critical values of t in table for alpha = 0.05 (or 0.01, 0.001) with \((N_1 + N_2 - 2)\) degrees of freedom
- One-sided:
  - testing one mean is larger than other
    - E.g., for (alpha=0.05, \(N_1 = N_2 = 10\)): \(t = 1.734\)
- Two-sided:
  - testing means are different
    - E.g., for (alpha=0.05, \(N_1 = N_2 = 10\)): \(t = 2.101\)