CS5740: Natural Language Processing

Text Classification

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Slides adapted from Dan Klein, Dan Jurafsky, Chris Manning, Michael Collins, Luke Zettlemoyer, Yejin Choi, and Slav Petrov
Overview

• Classification Problems
  – Spam vs. Non-spam, Text Genre, Word Sense, etc.

• Supervised Learning
  – Naïve Bayes
  – Log-linear models (Maximum Entropy Models)
  – Weighted linear models and the Perceptron
  – Neural networks
Supervised Learning: Data

• Learning from annotated data
• Often the biggest problem
• Why?
  – Annotation requires specific expertise
  – Annotation is expensive
  – Data is private and not accessible
  – Often difficult to define and be consistent
• Always think about the data, and how much of it your model needs (even better: think of the data first, model second)
Held-out Data

• Important tool for estimating generalization:
  – Train on one set, and evaluate during development on another
  – Test data: only use once!
Classification

• Automatically make a decision about inputs
  – Example: document \( \rightarrow \) category
  – Example: image of digit \( \rightarrow \) digit
  – Example: image of object \( \rightarrow \) object type
  – Example: query + webpage \( \rightarrow \) best match
  – Example: symptoms \( \rightarrow \) diagnosis
  – ...

• Three main ideas:
  – Representation as feature vectors
  – Scoring by linear functions
  – Learning by optimization
Example: Spam Filter

• Input: email
• Output: spam/ham
• Setup:
  – Get a large collection of example emails, each labeled “spam” or “ham”
  – Note: someone has to hand label all this data!
  – Goal: learn to predict labels of new, future emails
• Features: The attributes used to make the ham / spam decision
  – Words: FREE!
  – Text Patterns: $dd, CAPS
  – Non-text: SenderInContacts
  – …

Dear Sir.

First, I must solicit your confidence in this transaction, this is by virtue of its nature as being utterly confidencial and top secret. …

TO BE REMOVED FROM FUTURE MAILINGS, SIMPLY REPLY TO THIS MESSAGE AND PUT "REMOVE" IN THE SUBJECT.

99 MILLION EMAIL ADDRESSES FOR ONLY $99

Ok, I know this is blatantly OT but I'm beginning to go insane. Had an old Dell Dimension XPS sitting in the corner and decided to put it to use, I know it was working pre being stuck in the corner, but when I plugged it in, hit the power nothing happened.
More Examples

• Variants of spam
  – Comments, email
• Abuse and harassment
• Fake reviews
• Simple intent ("siri, what is the time?")
• Topic (for news) and domain (for papers)
• Medical
  – Disease, surgical procedure, insurance code
Probabilistic Classifiers

- Two broad approaches to predicting a class $y^*$

- Joint / Generative (e.g., Naïve Bayes)
  - Work with a \textit{joint} probabilistic model of the data
  - Assume functional form for $P(X|y)$, $P(y)$
  - Estimate probabilities from data
  - Use Bayes’ rule to calculate $P(y|X)$
  - E.g., represent $p(y,X)$ as Naïve Bayes model, compute $y^* = \arg\max_y p(y)p(X|y)$
  - \textbf{Advantages:} learning weights is easy and well understood

- Conditional / Discriminative (e.g., Logistic Regression)
  - Work with \textit{conditional} probability $p(y|X)$
  - We can then directly compute $y^* = \arg\max_y p(y|X)$
  - Estimate parameters from data
  - \textbf{Advantages:} Don’t have to model $p(X)$! Can develop \textit{feature rich} models for $p(y|X)$
Text Categorization

- Goal: classify documents into broad semantic topics

Obama is hoping to rally support for his $825 billion stimulus package on the eve of a crucial House vote. Republicans have expressed reservations about the proposal, calling for more tax cuts and less spending. GOP representatives seemed doubtful that any deals would be made.

California will open the 2009 season at home against Maryland Sept. 5 and will play a total of six games in Memorial Stadium in the final football schedule announced by the Pac-10 Conference Friday. The original schedule called for 12 games over 12 weekends.

- Which one is the politics document? (And how much deep processing did that decision take?)
- First approach: bag-of-words and Naïve-Bayes models
- More approaches later...
- Usually begin with a labeled corpus containing examples of each class
General Text Classification

• Input:
  – Document $X$ of length $|X|$ is a sequence of tokens:

\[ X = \langle x_1, \ldots, x_{|X|} \rangle \]

• Output:
  – One of $k$ labels $y$
Naïve-Bayes Models

- Generative model: pick a topic, then generate a document
- Naïve-Bayes assumption:
  - All words are independent given the topic.

\[
p(y, X) = q(y) \prod_{i=1}^{\mid X \mid} q(x_i \mid y)
\]

\[
\begin{align*}
&y \\
&\x_1 \quad \x_2 \quad \ldots \quad \x_n
\end{align*}
\]
Using NB for Classification

- We have a joint model of topics and documents
  \[ p(y, X) = q(y) \prod_{i=1}^{\mid X \mid} q(x_i \mid y) \]
- To assign a label \( y^* \) to a new document \( \langle x_1, x_2, \ldots, x_n \rangle \):
  \[ y^* = \arg \max_y p(y, X) = \arg \max_y q(y) \prod_{i=1}^{\mid X \mid} q(x_i \mid y) \]
Learning: Maximum Likelihood Estimate (MLE)

\[ p(y, X) = q(y) \prod_{i=1}^{\|X\|} q(x_i \mid y) \]

- Parameters to estimate:
  - \( q(y) = \theta_y \) for each topic \( y \)
  - \( q(x \mid y) = \theta_{xy} \) for each topic \( y \) and word \( x \)
- Data:
  \[ \{(X^{(j)}, y^{(j)})\}_{j=1}^{N} \]
- Objective:
  \[ \arg \max_{\theta} \prod_{j=1}^{N} p(y^{(j)}, X^{(j)}) = \arg \max_{\theta} \prod_{j=1}^{N} q(y^{(j)}) \prod_{i=1}^{\|X^{(j)}\|} q(x_i \mid y^{(j)}) \]
MLE

\[ p(y, X) = q(y) \prod_{i=1}^{\left| X \right|} q(x_i \mid y) \]

\[
\begin{align*}
\text{arg max} & \prod_{j=1}^{N} p(y^{(j)}, X^{(j)}) = \arg \max_{\theta} \prod_{j=1}^{N} q(y^{(j)}) \prod_{i=1}^{\left| X^{(j)} \right|} q(x_i \mid y^{(j)})
\end{align*}
\]

• How do we do learning? We count!

\[
q(y) = \theta_y = \frac{C(y)}{N} \quad q(x \mid y) = \theta_{xy} = \frac{C(x, y)}{C(y)}
\]
Word Sparsity

\[ q(y) = \theta_y = \frac{C(y)}{N} \quad q(x \mid y) = \theta_{xy} = \frac{C(x, y)}{C(y)} \]
Using NB for Classification

• We have a joint model of topics and documents

\[ p(y, X) = q(y) \prod_{i=1}^{|X|} q(x_i \mid y) \]

• To assign a label \( y^* \) to a new document \( \langle x_1, x_2, ..., x_n \rangle \):

\[ y^* = \arg \max_y p(y, X) = \arg \max_y q(y) \prod_{i=1}^{|X|} q(x_i \mid y) \]

• We get \( q(x_i \mid y) = 0 \) when \( C(x_i, y) = 0 \)

• Solution: smoothing + handling unknown words
  – More when we discuss language models
Example: Word-sense Disambiguation

• Example:
  – living plant vs. manufacturing plant

• How do we tell these senses apart?
  – “context”

The plant which had previously sustained the town’s economy shut down after an extended labor strike. The plants at the entrance, dry and wilted, the first victims of …
Case Study: Word Senses

• Words have multiple distinct meanings, or senses:
  – Plant: living plant, manufacturing plant, …
  – Title: name of a work, ownership document, form of address, material at the start of a film, …

• Many levels of sense distinctions
  – Homonymy: totally unrelated meanings
    • river bank, money bank
  – Polysemy: related meanings
    • star in sky, star on TV
  – Systematic polysemy: productive meaning extensions or metaphor
    • metonymy such as organizations to their buildings
  – Sense distinctions can be extremely subtle (or not)

• Granularity of senses needed depends a lot on the task

• Why is it important to model word senses?
  – Translation, parsing, information retrieval

Android vs. Apple
Word Sense Disambiguation

• Example: living plant vs. manufacturing plant
• How do we tell these senses apart?
  – “context”

  The plant which had previously sustained the town’s economy shut down after an extended labor strike. The plants at the entrance, dry and wilted, the first victims of …

  – Maybe it’s just text categorization
  – Each word sense represents a topic
  – Run a Naïve-Bayes classifier?

• Bag-of-words classification works OK for noun senses
  – 90% on classic, shockingly easy examples (line, interest, star)
  – 80% on senseval-1 nouns
  – 70% on senseval-1 verbs
Verb WSD

• Why are verbs harder?
  – Verbal senses less topical
  – More sensitive to structure, argument choice

• Verb Example: “Serve”
  – [function] The tree stump serves as a table
  – [enable] The scandal served to increase his popularity
  – [dish] We serve meals for the homeless
  – [enlist] She served her country
  – [jail] He served six years for embezzlement
  – [tennis] It was Agassi’s turn to serve
  – [legal] He was served by the sheriff
Better Features

• There are smarter features:
  – Argument selectional preference:
    • serve NP[meals] vs. serve NP[papers] vs. serve NP[country]
  – Sub-categorization:
    • [function] serve PP[as]
    • [enable] serve VP[to]
    • [tennis] serve <intransitive>
    • [food] serve NP {PP[to]}
  – Can be captured poorly (but robustly) with modified Naïve Bayes approach

• Other constraints (Yarowsky 95)
  – One-sense-per-discourse (only true for broad topical distinctions)
  – One-sense-per-collocation (pretty reliable when it kicks in: manufacturing plant, flowering plant)
Complex Features with NB

• Example: Washington County jail served 11,166 meals last month - a figure that translates to feeding some 120 people three times daily for 31 days.

• So we have a decision to make based on a set of cues:
  – context:jail, context:county, context:feeding, …
  – local-context:jail, local-context:meals
  – subcat:NP, direct-object-head:meals

• Not clear how build a generative derivation for these:
  – Choose topic, then decide on having a transitive usage, then pick “meals” to be the object’s head, then generate other words?
  – How about the words that appear in multiple features?
  – Hard to make this work (though maybe possible)
  – No real reason to try
Where we are?

• So far: Naïve Bayes models for classification
  – Generative models, estimating $P(X \mid y)$ and $P(y)$
  – Assumption: features are independent given the label (often violated in practice)
  – Easy to estimate (just count!)

• Next: Discriminative models
  – Estimating $P(y \mid X)$ directly
  – Very flexible feature handling
  – Require numerical optimization methods
A Discriminative Approach

• View WSD as a discrimination task, directly estimate:

\[ P(\text{sense} \mid \text{context:} \text{jail, context:county, context:feeding, ... local-context:} \text{jail, local-context:meals subcat:NP, direct-object-head:meals, ....} ) \]

• Have to estimate multinomial (over senses) where there are a huge number of things to condition on

• Many feature-based classification techniques out there
  – Discriminative models extremely popular in the NLP community!
Feature Representations

Washington County jail served 11,166 meals last month - a figure that translates to feeding some 120 people three times daily for 31 days.

- Features are indicator functions which count the occurrences of certain patterns in the input.
- Initially: we will have different feature values for every pair of input $X$ and class $y$.

\[
\begin{align*}
\text{context:} & \text{jail} = 1 \\
\text{context:} & \text{county} = 1 \\
\text{context:} & \text{feeding} = 1 \\
\text{context:} & \text{game} = 0 \\
\text{local-context:} & \text{jail} = 1 \\
\text{local-context:} & \text{meals} = 1 \\
\text{local-context:} & \text{game} = 0 \\
\text{object-head:} & \text{meals} = 1 \\
\text{object-head:} & \text{ball} = 0 
\end{align*}
\]
Example: Text Classification

• Goal: classify document to categories
  
  … win the election …  POLITICS
  … win the game …  SPORTS
  … see a movie …  OTHER

• Classically: based on words in the document
• But other information sources are potentially relevant:
  – Document length
  – Average word length
  – Document’s source
  – Document layout
Some Notation

INPUT

\( X^{(j)} \) ... win the election ...

OUTPUT SPACE

\( \mathcal{Y} \) SPORTS, POLITICS, OTHER

OUTPUT

\( y \) SPORTS

TRUE OUTPUT

\( y^{(j)} \) POLITICS

FEATURE VECTOR

\( \phi(X^{(j)}, y) \) [1 0 1 0 0 0 0 0 0 0 0 0]

SPORSTS+”win” POLITICS+”win”
Block Feature Vectors

• Input has features, which are multiplied by outputs to form the candidates

\[ \phi(X, SPORTS) = [1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \]
\[ \phi(X, POLITICS) = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0] \]
\[ \phi(X, OTHER) = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0] \]
Non-block Feature Vectors

- Sometimes the features of candidates cannot be decomposed in this regular way
- Example: a parse tree’s features may be the rules used for sentence $X$

$$\phi(X, \begin{array}{c} \text{S} \\ \text{NP} \\ \text{VP} \end{array}) = [1 \ 0 \ 1 \ 0 \ 1]$$

$$\phi(X, \begin{array}{c} \text{S} \\ \text{NP} \\ \text{V} \\ \text{NP} \end{array}) = [1 \ 1 \ 0 \ 1 \ 0]$$

- Different candidates will often share features
- We’ll return to the non-block case later
**Linear Models: Scoring**

• In a linear model, each feature gets a weight in $w$

$$\phi(X, SPORTS) = [1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$\phi(X, POLITICS) = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0]$$

$$w = [1 \ 1 \ -1 \ -2 \ 1 \ -1 \ 1 \ -2 \ -2 \ -1 \ -1 \ 1]$$

• We compare $y$’s on the basis of their linear scores:

$$score(X, y; w) = w^\top \cdot \phi(X, y)$$

$$score(X, POLITICS; w) = 1 \times 1 + 1 \times 1 = 2$$
Linear Models: Prediction Rule

\[ w = [1 \ 1 \ -1 \ -2 \ 1 \ -1 \ 1 \ -2 \ -2 \ -1 \ -1 \ 1] \]

- The linear prediction rule:
  \[ \text{prediction}(X, w) = \arg \max_{y \in \mathcal{Y}} w^\top \phi(X, y) \]

\[ \phi(X, \text{SPORTS}) = [1 \ 0 \ 1 \ 0 \ldots] \quad \text{score}(X, \text{SPORTS}, w) = 1 \times 1 + (-1) \times 1 = 0 \]
\[ \phi(X, \text{POLITICS}) = [\ldots 1 \ 0 \ 1 \ 0 \ldots] \quad \text{score}(X, \text{POLITICS}, w) = 1 \times 1 + 1 \times 1 = 2 \]
\[ \phi(X, \text{OTHER}) = [\ldots 1 \ 0 \ 1 \ 0] \quad \text{score}(X, \text{OTHER}, w) = (-2) \times 1 + (-1) \times 1 = -3 \]

\[ \text{prediction}(X, w) = \text{POLITICS} \]
(Multinomial) Naïve-Bayes is a linear model:

\[ \phi(X, y) = [\ldots 0 \ldots, 1, \#v_1, \#v_2, \ldots, \#v_n, \ldots] \]
\[ w = [\ldots, \log P(y), \log P(v_1 | y), \log P(v_2 | y), \ldots, \log P(v_n | y), \ldots] \]

\[
\text{score}(X, y, w) = w^\top \phi(X, y) \\
= \log P(y) + \sum_{k=1}^{n} \#v_k \log P(v_k | y) \\
= \log \left( P(y) \prod_{k=1}^{n} P(v_k | y)^{\#v_k} \right) \\
= \log \left( P(y) \prod_{i=1}^{|X|} P(x_i | y) \right) \\
= \log P(X, y)
\]
How to Pick Weights?

• Goal: choose “best” vector $w$ given training data
  – Best for classification

• The ideal: the weights which have greatest test set accuracy / F1 / whatever
  – But, don’t have the test set
  – Must compute weights from training set

• Maybe we want weights which give best training set accuracy?
  – Hard optimization problem
  – May not (does not) generalize to test set
    • Easy to overfit
Maximum Entropy Models (MaxEnt)

- Maximum entropy (logistic regression)
  - Model: use the scores as probabilities:
    \[
    p(y | X; w) = \frac{\exp(w \cdot \phi(X, y))}{\sum_{y'} \exp(w \cdot \phi(X, y'))}
    \]
  - Learning: maximize the (log) conditional likelihood of training data \(\{(X^{(i)}, y^{(i)})\}_{i=1}^{N}\)
    \[
    L(w) = \log \prod_{i=1}^{N} p(y^{(i)} | X^{(i)}; w) = \sum_{i=1}^{N} \log p(y^{(i)} | X^{(i)}; w)
    \]
    \[
    w^* = \arg \max_w L(w)
    \]
  - Prediction:
    \[
    y^* = \arg \max_y p(y | X; w)
    \]
Unconstrained Optimization

\[ L(w) = \sum_{i=1}^{N} \log P(y^{(i)} | X^{(i)}; w) \quad w^* = \arg \max_w L(w) \]

- Unfortunately, \( \arg \max_w L(w) \) doesn’t have a close formed solution
- The MaxEnt objective is an unconstrained optimization problem

- Basic idea: move uphill from current guess
- Gradient ascent / descent follows the gradient incrementally
- At local optimum, derivative vector is zero
- Will converge if step sizes are small enough, but not efficient
- All we need is to be able to evaluate the function and its derivative
Unconstrained Optimization

• Once we have a function $f$, we can find a local optimum by iteratively following the gradient

• For convex functions:
  – A local optimum will be global
  – Does this mean that all is good?

• Basic gradient ascent isn’t very efficient, but there are simple enhancements which take into account previous gradients: conjugate gradient, L-BFGS

• There are special-purpose optimization techniques for MaxEnt, like iterative scaling, but they aren’t better
Derivative of the MaxEnt Objective

\[ L(w) = \sum_{i=1}^{N} \log p(y^{(i)} \mid X^{(i)}; w) \quad p(y \mid X; w) = \frac{e^{w \cdot \phi(X,y)}}{\sum_{y'} e^{w \cdot \phi(X,y')}} \]

• Some necessities:

\[ w \cdot \phi(x, y) = w_1 \times \phi_1(x, y) + w_2 \times \phi_2(x, y) + \cdots + w_n \times \phi_n(x, y) \]

\[ \frac{\partial}{\partial x} \log_a u = \frac{1}{u \log_e a} \frac{\partial}{\partial x} u \quad \frac{\partial}{\partial x} e^u = e^u \frac{\partial}{\partial x} u \]

\[ \frac{\partial}{\partial x} \log_e u = \frac{1}{u \log_e e} \frac{\partial}{\partial x} u = \frac{1}{u} \frac{\partial}{\partial x} u \]
Derivative of the MaxEnt Objective

\[ L(w) = \sum_{i=1}^{N} \log p(y^{(i)} \mid X^{(i)}; w) \]

\[
\frac{\partial}{\partial w_j} L(w) = \frac{\partial}{\partial w_j} \sum_{i=1}^{N} \log P(y^{(i)} \mid X^{(i)}; w)
\]

\[
= \frac{\partial}{\partial w_j} \sum_{i=1}^{N} \log \left( \frac{e^{w \cdot \phi(X^{(i)}, y^{(i)})}}{\sum_{y'} e^{w \cdot \phi(X^{(i)}, y')}} \right)
\]

\[
= \frac{\partial}{\partial w_j} \sum_{i=1}^{N} \left( \log e^{w \cdot \phi(X^{(i)}, y^{(i)})} - \log \sum_{y'} e^{w \cdot \phi(X^{(i)}, y')} \right)
\]

\[
= \frac{\partial}{\partial w_j} \sum_{i=1}^{N} \left( w \cdot \phi(X^{(i)}, y^{(i)}) - \log \sum_{y'} e^{w \cdot \phi(X^{(i)}, y')} \right)
\]

\[
= \sum_{i=1}^{N} \left( \phi_j(X^{(i)}, y^{(i)}) - \frac{1}{\sum_{y'} e^{w \cdot \phi(X^{(i)}, y')}} \sum_{y'} e^{w \cdot \phi(X^{(i)}, y')} \phi_j(X^{(i)}, y') \right)
\]

\[
= \sum_{i=1}^{N} \left( \phi_j(X^{(i)}, y^{(i)}) - \sum_{y'} \frac{e^{w \cdot \phi(X^{(i)}, y')}}{\sum_{y''} e^{w \cdot \phi(X^{(i)}, y'')}} \phi_j(X^{(i)}, y') \right)
\]

\[
= \sum_{i=1}^{N} \left( \phi_j(X^{(i)}, y^{(i)}) - \sum_{y'} P(y' \mid X^{(i)}; w) \phi_j(X^{(i)}, y') \right)
\]
Derivative of the MaxEnt Objective

\[ L(w) = \sum_{i=1}^{N} \log p(y^{(i)} | X^{(i)}; w) \quad p(y | X; w) = \frac{e^{w \cdot \phi(X,y)}}{\sum_{y'} e^{w \cdot \phi(X,y')}} \]

\[
\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^{N} \left( \phi_j(X^{(i)}, y^{(i)}) - \sum_{y'} P(y' | X^{(i)}; w) \phi_j(X^{(i)}, y') \right)
\]

Total count of feature \( j \) in correct candidates

Expected count of feature \( j \) in predicted candidates
Expected Counts

- The optimum parameters are the ones for which each feature’s predicted expectation equals its empirical expectation.

\[
\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^{N} \left( \phi_j(X^{(i)}, y^{(i)}) - \sum_{y'} P(y'|X^{(i)}; w) \phi_j(X^{(i)}, y') \right)
\]

<table>
<thead>
<tr>
<th>Context-word: jail + cat: prison</th>
<th>Actual Counts</th>
<th>Empirical Counts</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.7</td>
<td>+0.3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Context-word: jail + cat: food</th>
<th>Actual Counts</th>
<th>Empirical Counts</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.3</td>
<td>-0.3</td>
<td></td>
</tr>
</tbody>
</table>
A Very Nice Objective

• The MaxEnt objective behaves nicely:
  – Differentiable (so many ways to optimize)
  – Convex (so no local optima)

Convexity guarantees a **single, global maximum value** because any higher points are greedily reachable.
What About Overfitting?

• For Naïve Bayes, we were worried about zero counts in MLE estimates
  – Can that happen here?

• Regularization (smoothing) for Log-linear models
  – Instead, we worry about large feature weights
  – Add a L2 regularization term to the likelihood to push weights towards zero

\[ L(w) = \sum_{i=1}^{N} \log p(y^{(i)}|X^{(i)}; w) - \frac{\lambda}{2} ||w||^2 \]
Derivative of the Regularized MaxEnt Objective

- Unfortunately, $\text{argmax}_w L(w)$ still doesn’t have a close formed solution
- We will have to differentiate and use gradient ascent

$$L(w) = \sum_{i=1}^{N} \left( w \cdot \phi(X^{(i)}, y^{(i)}) - \log \sum_y \exp(w \cdot \phi(X^{(i)}, y)) \right) - \frac{\lambda}{2} ||w||^2$$

$$\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^{N} \left( \phi_j(X^{(i)}, y^{(i)}) - \sum_y p(y|X^{(i)}; w) \phi_j(X^{(i)}, y) \right) - \lambda w_j$$

- Total count of feature j in correct candidates
- Expected count of feature j in predicted candidates
- Big weights are bad
Because of regularization, the more common prefixes have larger weights even though entire-word features are more specific.
Learning Classifiers

• Two probabilistic approaches to predicting classes $y^*$
  – **Joint**: work with a *joint* probabilistic model of the data, weights are (often) local conditional probabilities
    • E.g., represent $p(y,x)$ as Naïve Bayes model, compute $y^* = \arg\max_y p(y,X)$
  – **Conditional**: work with *conditional* probability $p(y \mid X)$
    • We can then direct compute $y^* = \arg\max_y p(y \mid X)$ Can develop *feature rich* models for $p(y \mid X)$.

• But, why estimate a distribution at all?
  – Linear predictor: $y^* = \arg\max_y w \cdot \phi(X,y)$
  – Perceptron algorithm
    • Online (or batch)
    • Error driven
    • Simple, additive updates
NEW NAVY DEVICE LEARNS BY DOING

Psychologist Shows Embryo of Computer Designed to Read and Grow Wiser

WASHINGTON, July 7 (UPI) — The Navy revealed the embryo of an electronic computer today that it expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence.

The Navy said the perception would be the first computer that is capable of receiving, recognizing and identifying its surroundings without any human control or instruction. The perception will also be able to learn and distinguish right from left after fifty attempts in the Navy's demonstration for newsmen.

The embryo—the Weather Bureau's $2,000,000 "704" computer—learned to differentiate between right and left after fifty attempts in the Navy's demonstration for newsmen.

The service said it would use this principle to build the first of its Perceptron thinking machines that will be able to read and write. It is expected to be finished in about a year at a cost of $100,000.

Dr. Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechanical space explorers.
Perceptron Learning

• The perceptron algorithm
  – Iteratively processes the training set, reacting to training errors
  – Can be thought of as trying to drive down training error

• The online (binary $\rightarrow y = \pm 1$) perceptron algorithm:
  – Start with zero weights
  – Visit training instances $(X^{(i)}, y^{(i)})$ one by one, until all correct
    • Make a prediction
      \[ y^* = \text{sign}(w \cdot \phi(X^{(i)})) \]
      • If correct ($y^* = y^{(i)}$): no change, goto next example!
      • If wrong: adjust weights
        \[ w = w - y^* \phi(X^{(i)}) \]
Two Simple Examples

Data set I:

\[ X^{(1)} = [1, 1], \quad y^{(1)} = 1 \]
\[ X^{(2)} = [1, -1], \quad y^{(2)} = 1 \]
\[ X^{(3)} = [-1, -1], \quad y^{(3)} = -1 \]

Data set II:

\[ X^{(1)} = [1, 1], \quad y^{(1)} = 1 \]
\[ X^{(2)} = [1, -1], \quad y^{(2)} = 1 \]
\[ X^{(3)} = [-1, -1], \quad y^{(3)} = -1 \]
\[ X^{(4)} = [0.25, 0.25], \quad y^{(4)} = -1 \]
Separating Hyperplane

- The perceptron finds a separating hyperplane

\[ X^{(1)} = [1, 1], \quad y^{(1)} = 1 \]
\[ X^{(2)} = [1, -1], \quad y^{(2)} = 1 \]
\[ X^{(3)} = [-1, -1], \quad y^{(3)} = -1 \]

\[ w = [1, 1] \]

Finding the hyperplane:

\[ w \cdot [x, y] = 1 \times x + 1 \times y = 0 \]
Separable Case
Geometric Interpretation II

- Start with zero weights
- Visit training instances \((X^{(i)}, y^{(i)})\) one by one, until all correct
  - Make a prediction
    \[ y^* = \text{sign}(w \cdot \phi(X^{(i)})) \]
  - If correct \((y^* = y^{(i)})\): no change, goto next example!
  - If wrong: adjust weights
    \[ w = w - y^* \phi(X^{(i)}) \]

\[ y^* = 1, y^{(i)} = -1 \]
Geometric Interpretation

• The perceptron finds a separating hyperplane

\[ X^{(1)} = [1, 1], \quad y^{(1)} = 1 \]
\[ X^{(2)} = [1, -1], \quad y^{(2)} = 1 \]
\[ X^{(3)} = [-1, -1], \quad y^{(3)} = -1 \]
\[ X^{(4)} = [0.25, 0.25], \quad y^{(4)} = -1 \]
\[ w = [0, 0] \]
\[ w = [1, 1] \]
\[ w = [0.75, 0.75] \]
\[ w = [0.5, 0.5] \]
\[ w = [0.25, 0.25] \]

Is there a separating hyperplane?
Quick Trick: Adding Bias

• Decision rule:

\[ y^* = \text{sign}(w \cdot \phi(X^{(i)}) + b) \]

• Algorithm stays the same!

• Only difference: dummy always-on feature

\[ X^{(1)} = [1, 1], \quad y^{(1)} = 1 \]
\[ X^{(2)} = [1, -1], \quad y^{(2)} = 1 \]
\[ X^{(3)} = [-1, -1], \quad y^{(3)} = -1 \]

\[ w = [0, 0] \in \mathbb{R}^2 \]
Multiclass Perceptron

- If we have multiple classes:
  - A weight vector for each class: \( w_y \)
  - Score (activation) of a class \( y \):
    \[ w_y \cdot \phi(X) \]
  - Prediction highest score wins
    \[ y^* = \arg \max_y w_y \cdot \phi(X) \]
Multiclass Perceptron

- Start with zero weights
- Visit training instances \((X^{(i)}, y^{(i)})\) one by one
  - Make a prediction
    \[
    y^* = \arg\max_y w_y \cdot \phi(X^{(i)})
    \]
  - If correct \((y^* = y^{(i)})\): no change, continue!
  - If wrong: adjust weights
    \[
    w_{y^{(i)}} = w_{y^{(i)}} + \phi(X^{(i)})
    \]
    \[
    w_{y^*} = w_{y^*} - \phi(X^{(i)})
    \]
Multiclass Perceptron: Rewrite

- Compare all possible outputs
  - Highest score wins
  - Approximate visualization
    (usually hard)

\[ y^* = \arg \max_y w \cdot \phi(X, y) \]
Perceptron Learning

• Start with zero weights
• Visit training instances \( (X^{(i)}, y^{(i)}) \) one by one
  – Make a prediction
    \[
    y^* = \arg \max_y \ w \cdot \phi(X^{(i)}, y)
    \]
  – If correct \( y^* = y^{(i)} \): no change, goto next example!
  – If wrong: adjust weights
    \[
    w = w + \phi(X^{(i)}, y^{(i)}) - \phi(X^{(i)}, y^*)
    \]
From MaxEnt to the Perceptron

• Prediction: \( y^* = \arg \max_y w \cdot \phi(x^{(i)}, y) \)

• Update: \( w = w + \phi(x^{(i)}, y^{(i)}) - \phi(x^{(i)}, y^*) \)

• MaxEnt gradient for \( x_i \):

\[
\frac{\partial}{\partial w_j} L(w) = \phi_j(x^{(i)}, y^{(i)}) - \sum_{y'} P(y'|x^{(i)}; w) \phi_j(x^{(i)}, y')
\]

Approximate expectation with max!

\[
\approx \phi_j(x^{(i)}, y^{(i)}) - \phi_j(x^{(i)}, y^*)
\]

where \( y^* = \arg \max_y w \cdot \phi_j(x^{(i)}, y) \)
Perceptron Learning

- No counting or computing probabilities on training set
- **Separability**: some parameters get the training set perfectly correct
- **Convergence**: if the training is separable, perceptron will eventually converge
- **Mistake Bound**: the maximum number of mistakes (binary case) related to the margin or degree of separability
Problems with the Perceptron

- **Noise**: if the data isn’t separable, weights might thrash
  - Averaging weight vectors over time can help (averaged perceptron)
- **Mediocre generalization**: finds a “barely” separating solution
- **Overtraining**: test / held-out accuracy usually rises, then falls
  - Overtraining is a kind of overfitting
Perceptron Example

Drugs
- Apo-Loperamide
- Minims Tropicamide
- Mexate
- Maxair

Name
- Alexander
- Anders
- Frederick
- Donald

Please use 2 features at most!
## Perceptron Example

<table>
<thead>
<tr>
<th>Drugs</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apo-Loperamide</td>
<td>Alexander</td>
</tr>
<tr>
<td>Minims Tropicamide</td>
<td>Anders</td>
</tr>
<tr>
<td>Mexate</td>
<td>Frederick</td>
</tr>
<tr>
<td>Maxair</td>
<td>Donald</td>
</tr>
<tr>
<td>Tebamide</td>
<td>Roderick</td>
</tr>
<tr>
<td>Dexedrine</td>
<td>Malcolm</td>
</tr>
</tbody>
</table>

Train | Test
Three Views of Classification

• Naïve Bayes:
  – Parameters from data statistics
  – Parameters: probabilistic interpretation
  – Training: one pass through the data

• Log-linear models:
  – Parameters from gradient ascent
  – Parameters: linear, probabilistic model, and discriminative
  – Training: gradient ascent (usually batch), regularize to stop overfitting

• The Perceptron:
  – Parameters from reactions to mistakes
  – Parameters: discriminative interpretation
  – Training: go through the data until validation accuracy maxes out
A Note on Features: TF/IDF

• More frequent terms in a document are more important:

\[ f_{ij} = \text{frequency of term } i \text{ in document } j \]

• May want to normalize term frequency (tf) by dividing by the frequency of the most common term in the document:

\[ tf_{ij} = \frac{f_{ij}}{\max_i \{ f_{ij} \}} \]

• Terms that appear in many different documents are less indicative:

\[ df_i = \text{document frequency of term } i = \text{number of documents containing term } i \]

\[ idf_i = \text{inverse document frequency of term } i = \log_2(\frac{N}{df_i}) \]

\[ N = \text{total number of documents} \]

• An indication of a term’s discrimination power
• Log used to dampen the effect relative to tf
• A typical combined term important indicator is tf-idf weighting

\[ w_{ij} = tf_{ij} idf_i = tf_{ij} \log_2(\frac{N}{df_i}) \]