Sequence Prediction and Part-of-speech Tagging

Instructor: Yoav Artzi
Overview

• POS Tagging: the problem
• Hidden Markov Models (HMM)
  – Supervised Learning
  – Inference
    • The Viterbi algorithm
• Feature-rich models
  – Maximum-entropy Markov Models
  – Perceptron
  – Conditional Random Fields
# Parts of Speech

## Open class (lexical) words

<table>
<thead>
<tr>
<th>Nouns</th>
<th>Verbs</th>
<th>Adjectives</th>
<th>Adverbs</th>
<th>Numbers</th>
<th>Prepositions</th>
<th>Particles</th>
<th>Interjections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proper</td>
<td>Main</td>
<td>old</td>
<td>slowly</td>
<td>… more</td>
<td>to</td>
<td>off</td>
<td>Ow</td>
</tr>
<tr>
<td>IBM</td>
<td>can</td>
<td>older</td>
<td></td>
<td></td>
<td>with</td>
<td>up</td>
<td>Eh</td>
</tr>
<tr>
<td>Italy</td>
<td>had</td>
<td>oldest</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common</td>
<td>see</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cat / cats</td>
<td>registered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>snow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Closed class (functional)

<table>
<thead>
<tr>
<th>Determiners</th>
<th>Conjunctions</th>
<th>Pronouns</th>
</tr>
</thead>
<tbody>
<tr>
<td>the some</td>
<td>and or</td>
<td>he its</td>
</tr>
</tbody>
</table>

---

IBM
Italy
cat / cats
snow

... more

Eh
Ow

122,312
one

... more
POS Tagging

- Words often have more than one POS: \textit{back}
  - The \textit{back} door = JJ
  - On my \textit{back} = NN
  - Win the voters \textit{back} = RB
  - Promised to \textit{back} the bill = VB

- The POS tagging problem is to determine the POS tag for a particular instance of a word.
POS Tagging

- Input: Plays well with others
- Ambiguity: NNS/VBZ UH/JJ/NN/RB IN NNS
- Output: Plays/VBZ well/RB with/IN others/NNS
- Uses:
  - Text-to-speech (how do we pronounce “lead”?)
  - Can write regular expressions like (Det) Adj* N+ over the output for phrases, etc.
  - As input to or to speed up a full parser
  - If you know the tag, you can back off to it in other tasks
Penn TreeBank Tagset

- Possible tags: 45
- Tagging guidelines: 36 pages
- Newswire text
<table>
<thead>
<tr>
<th>Tag</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC</td>
<td>conjunction, coordinating</td>
<td>mid-1890, nine-thirty, 0.5, one</td>
</tr>
<tr>
<td>CD</td>
<td>numeral, cardinal</td>
<td>a, all, every, no, that, the</td>
</tr>
<tr>
<td>DT</td>
<td>determiner</td>
<td>there</td>
</tr>
<tr>
<td>EX</td>
<td>existential</td>
<td>there</td>
</tr>
<tr>
<td>FW</td>
<td>foreign word</td>
<td>gemeinschaft, hund, ich, jeux</td>
</tr>
<tr>
<td>IN</td>
<td>preposition or conjunction, subordinating</td>
<td>among, whether, out, on, by, if</td>
</tr>
<tr>
<td>JJ</td>
<td>adjective or numeral, ordinal</td>
<td>third, ill-mannered, regrettable</td>
</tr>
<tr>
<td>JJR</td>
<td>adjective, comparative</td>
<td>braver, cheaper, taller</td>
</tr>
<tr>
<td>JJS</td>
<td>adjective, superlative</td>
<td>bravest, cheapest, tallest</td>
</tr>
<tr>
<td>MD</td>
<td>modal auxiliary</td>
<td>can, may, might, will, would</td>
</tr>
<tr>
<td>NN</td>
<td>noun, common, singular or mass</td>
<td>cabbage, thermostat, investment, subhumanity</td>
</tr>
<tr>
<td>NNP</td>
<td>noun, proper, singular</td>
<td>Motown, Cougar, Yvette, Liverpool</td>
</tr>
<tr>
<td>NNPS</td>
<td>noun, proper, plural</td>
<td>Americans, Materials, States</td>
</tr>
<tr>
<td>NNS</td>
<td>noun, common, plural</td>
<td>undergraduates, bric-a-brac, averages</td>
</tr>
<tr>
<td>POS</td>
<td>genitive marker</td>
<td>'s</td>
</tr>
<tr>
<td>PRP</td>
<td>pronoun, personal</td>
<td>hers, himself, it, we, them</td>
</tr>
<tr>
<td>PRP$</td>
<td>pronoun, possessive</td>
<td>her, his, mine, my, our, ours, their, thy, your</td>
</tr>
<tr>
<td>RB</td>
<td>adverb</td>
<td>occasionally, maddeningly, adventurously</td>
</tr>
<tr>
<td>RBR</td>
<td>adverb, comparative</td>
<td>further, gloomier, heavier, less-perfectly</td>
</tr>
<tr>
<td>RBS</td>
<td>adverb, superlative</td>
<td>best, biggest, nearest, worst</td>
</tr>
<tr>
<td>RP</td>
<td>particle</td>
<td>aboard, away, back, by, on, open, through</td>
</tr>
<tr>
<td>TO</td>
<td>&quot;to&quot; as preposition or infinitive marker</td>
<td>to</td>
</tr>
<tr>
<td>UH</td>
<td>interjection</td>
<td>huh, howdy, uh, whammo, shucks, heck</td>
</tr>
<tr>
<td>VB</td>
<td>verb, base form</td>
<td>ask, bring, fire, see, take</td>
</tr>
<tr>
<td>VBD</td>
<td>verb, past tense</td>
<td>pleaded, swiped, registered, saw</td>
</tr>
<tr>
<td>VBG</td>
<td>verb, present participle or gerund</td>
<td>stirring, focusing, approaching, erasing</td>
</tr>
<tr>
<td>VBN</td>
<td>verb, past participle</td>
<td>dilapidated, imitated, reunified, unsettled</td>
</tr>
<tr>
<td>VBP</td>
<td>verb, present tense, not 3rd person singular</td>
<td>twist, appear, comprise, mold, postpone</td>
</tr>
<tr>
<td>VBZ</td>
<td>verb, present tense, 3rd person singular</td>
<td>bases, reconstructs, marks, uses</td>
</tr>
<tr>
<td>WDT</td>
<td>WH-determiner</td>
<td>that, what, whatever, which, whichever</td>
</tr>
<tr>
<td>WP</td>
<td>WH-pronoun</td>
<td>that, what, whatever, which, who, whom</td>
</tr>
<tr>
<td>WP$</td>
<td>WH-pronoun, possessive</td>
<td>whose</td>
</tr>
<tr>
<td>WRB</td>
<td>Wh-adverb</td>
<td>however, whenever, where, why</td>
</tr>
</tbody>
</table>
Penn TreeBank Tagset

• How accurate are taggers? (Tag accuracy)
  – About 97% currently
  – But baseline is already 90%
    • Baseline is performance of simplest possible method
      – Tag every word with its most frequent tag
      – Tag unknown words as nouns
  – Partly easy because
    • Many words are unambiguous
    • You get points for them (the, a, etc.) and for punctuation marks!
  – Upperbound: probably 2% annotation errors
Hard Cases are Hard

- Mrs/NNP Shaefer/NNP never/RB got/VBD around/RP to/TO joining/VBG

- All/DT we/PRP gotta/VBN do/VB is/VBZ go/VB around/IN the/DT corner/NN

- Chateau/NNP Petrus/NNP costs/VCZ around/RB 250/CD
How Difficult is POS Tagging?

• About 11% of the word types in the Brown corpus are ambiguous with regard to part of speech
• But they tend to be very common words. E.g., *that*
  – I know *that* he is honest = IN
  – Yes, *that* play was nice = DT
  – You can’t go *that* far = RB
• 40% of the word tokens are ambiguous
The Tagset

• Wait, do we really need all these tags?
• What about other languages?
  – Each language has its own tagset
Tagsets in Different Languages

<table>
<thead>
<tr>
<th>Language</th>
<th>Source</th>
<th># Tags</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arabic</td>
<td>PADT/CoNLL07 (Hajič et al., 2004)</td>
<td>21</td>
</tr>
<tr>
<td>Basque</td>
<td>Basque3LB/CoNLL07 (Aduriz et al., 2003)</td>
<td>64</td>
</tr>
<tr>
<td>Bulgarian</td>
<td>BTB/CoNLL06 (Simov et al., 2002)</td>
<td>54</td>
</tr>
<tr>
<td>Catalan</td>
<td>CESS-ECE/CoNLL07 (Martí et al., 2007)</td>
<td>54</td>
</tr>
<tr>
<td>Chinese</td>
<td>Penn ChineseTreebank 6.0 (Palmer et al., 2007)</td>
<td>34</td>
</tr>
<tr>
<td>Chinese</td>
<td>Sinica/CoNLL07 (Chen et al., 2003)</td>
<td>294</td>
</tr>
<tr>
<td>Czech</td>
<td>PDT/CoNLL07 (Böhmová et al., 2003)</td>
<td>63</td>
</tr>
<tr>
<td>Danish</td>
<td>DDT/CoNLL06 (Kromann et al., 2003)</td>
<td>25</td>
</tr>
<tr>
<td>Dutch</td>
<td>Alpino/CoNLL06 (Van der Beek et al., 2002)</td>
<td>12</td>
</tr>
<tr>
<td>English</td>
<td>PennTreebank (Marcus et al., 1993)</td>
<td>45</td>
</tr>
<tr>
<td>French</td>
<td>FrenchTreebank (Abeillé et al., 2003)</td>
<td>30</td>
</tr>
<tr>
<td>German</td>
<td>Tiger/CoNLL06 (Brants et al., 2002)</td>
<td>54</td>
</tr>
<tr>
<td>German</td>
<td>Negra (Skut et al., 1997)</td>
<td>54</td>
</tr>
<tr>
<td>Greek</td>
<td>GDT/CoNLL07 (Prokopidis et al., 2005)</td>
<td>38</td>
</tr>
<tr>
<td>Hungarian</td>
<td>Szeged/CoNLL07 (Csendes et al., 2005)</td>
<td>43</td>
</tr>
<tr>
<td>Italian</td>
<td>ISST/CoNLL07 (Montemagni et al., 2003)</td>
<td>28</td>
</tr>
<tr>
<td>Japanese</td>
<td>Verbmobil/CoNLL06 (Kawata and Bartels, 2000)</td>
<td>80</td>
</tr>
<tr>
<td>Japanese</td>
<td>Kyoto4.0 (Kurohashi and Nagao, 1997)</td>
<td>42</td>
</tr>
<tr>
<td>Korean</td>
<td>Sejong (<a href="http://www.sejong.or.kr">http://www.sejong.or.kr</a>)</td>
<td>187</td>
</tr>
<tr>
<td>Portuguese</td>
<td>Floresta Sintá(c)tica/CoNLL06 (Afonso et al., 2002)</td>
<td>22</td>
</tr>
<tr>
<td>Russian</td>
<td>SynTagRus-RNC (Boguslavsky et al., 2002)</td>
<td>11</td>
</tr>
<tr>
<td>Slovene</td>
<td>SDT/CoNLL06 (Džeroski et al., 2006)</td>
<td>29</td>
</tr>
<tr>
<td>Spanish</td>
<td>Ancora-Cast3LB/CoNLL06 (Civit and Martí, 2004)</td>
<td>47</td>
</tr>
<tr>
<td>Swedish</td>
<td>Talbanken05/CoNLL06 (Nivre et al., 2006)</td>
<td>41</td>
</tr>
<tr>
<td>Turkish</td>
<td>METU-Sabanci/CoNLL07 (Oflazer et al., 2003)</td>
<td>31</td>
</tr>
</tbody>
</table>

[Petrov et al. 2012]
The Tagset

• Wait, do we really need all these tags?
• What about other languages?
  – Each language has its own tagset
    • But why is this bad?
    • Differences in downstream tasks
    • Harder to do language transfer
Alternative: The Universal Tagset

- 12 tags:
  - NOUN, VERB, ADJ, ADV, PRON, DET, ADP, NUM, CONJ, PRT, ‘.’, and X.
- Deterministic conversion from tagsets in 22 languages.
- Better unsupervised parsing results
- Was used to transfer parsers
Sources of Information

• What are the main sources of information for POS tagging?
  – Knowledge of neighboring words
    • Bill saw that man yesterday
    • NNP VB(D) DT NN NN
    • VB NN IN VB NN
  – Knowledge of word probabilities
    • *man* is rarely used as a verb….

• The latter proves the most useful, but the former also helps
Word-level Features

- Can do surprisingly well just looking at a word by itself:
  - Word the: the → DT
  - Lowercased words: importantly → RB
  - Prefixes unfathomable: un- → JJ
  - Suffixes Importantly: -ly → RB
  - Capitalization Meridian: CAP → NNP
  - Word shapes 35-year: d-x → JJ
Consider the problem of jointly modeling a pair of strings
– E.g.: part of speech tagging

The Georgia branch had taken on loan commitments …

The average of interbank offered rates plummeted …

Q: How do we map each word in the input sentence onto the appropriate label?
A: We can learn a joint distribution:

\[ p(x_1 \ldots x_n, y_1 \ldots y_n) \]

And then compute the most likely assignment:

\[ \arg \max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n) \]
Classic Solution: HMMs

We want a model of sequences $y$ and observations $x$

$$p(\ldots x_n, y_1 \ldots y_n) =$$

where $y_0 = \text{START}$ and we call $q(y_i | y_{i-1})$ the transition distribution and $e(x_i | y_i)$ the emission (or observation) distribution.
Model Assumptions

- Tag/state sequence is generated by a Markov model
- Words are chosen independently, conditioned only on the tag/state
- These are totally broken assumptions for POS: why?

$$p(x_1 \ldots x_n, y_1 \ldots y_n) = q(STOP | y_n) \prod_{i=1}^{n} q(y_i | y_{i-1}) e(x_i | y_i)$$
HMM for POS Tagging

The Georgia branch had taken on loan commitments …

- HMM Model:
  - States $Y =$
  - Observations $X =$
  - Transition dist’n $q(y_i | y_{i-1})$ models
  - Emission dist’n $e(x_i | y_i)$ models
HMM for POS Tagging

The Georgia branch had taken on loan commitments ...

DT     NNP        NN        VBD    VBN   RP   NN        NNS

• HMM Model:
  – States $Y = \{DT, NNP, NN, \ldots \}$ are the POS tags
  – Observations $X = V$ are words
  – Transition dist’n $q(y_i|y_{i-1})$ models the tag sequences
  – Emission dist’n $e(x_i|y_i)$ models words given their POS
HMM Inference and Learning

• Learning
  – Maximum likelihood: transitions \( q \) and emissions \( e \)
    \[
p(x_1 \ldots x_n, y_1 \ldots y_n) = q(\text{STOP}|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1}) e(x_i|y_i)
    \]

• Inference
  – Viterbi
    \[
y^* = \arg \max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n)
    \]
  – Forward backward
    \[
p(x_1 \ldots x_n, y_i) = \sum_{y_1 \ldots y_{i-1}} \sum_{y_{i+1} \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n)
    \]
Learning: Maximum Likelihood

\[ p(x_1 \ldots x_n, y_1 \ldots y_n) = q(STOP|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1})e(x_i|y_i) \]

- Maximum likelihood methods for estimating transitions \( q \) and emissions \( e \)

\[ q_{ML}(y_i|y_{i-1}) = \frac{c(y_{i-1}, y_i)}{c(y_{i-1})} \quad e_{ML}(x|y) = \frac{c(y, x)}{c(y)} \]

- Will these estimates be high quality?
  – Which is likely to be more sparse, \( q \) or \( e \)?
- Smoothing?
Learning: Low Frequency Words

\[ p(x_1 \ldots x_n, y_1 \ldots y_n) = q(STOP|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1})e(x_i|y_i) \]

- Typically, for transitions:
  - Linear Interpolation
    \[ q(y_i|y_{i-1}) = \lambda_1 q_{ML}(y_i|y_{i-1}) + \lambda_2 q_{ML}(y_i) \]
- However, other approaches used for emissions
  - **Step 1**: Split the vocabulary
    - Frequent words: appear more than \( M \) (often 5) times
    - Low frequency: everything else
  - **Step 2**: Map each low frequency word to one of a small, finite set of possibilities
    - For example, based on prefixes, suffixes, etc.
  - **Step 3**: Learn model for this new space of possible word sequences
Another Example: Chunking

• Goal: Segment text into spans with certain properties
• For example, named entities: PER, ORG, and LOC

Germany’s representative to the European Union’s veterinary committee Werner Zwingman said on Wednesday consumers should…

[Germany]_LOC’s representative to the [European Union]_ORG’s veterinary committee [Werner Zwingman]_PER said on Wednesday consumers should…

How is this a sequence tagging problem?
Named Entity Recognition

Germany ’s representative to the European Union ’s veterinary committee Werner Zwingman said on Wednesday consumers should…

[Germany]_{LOC} ’s representative to the [European Union]_{ORG} ’s veterinary committee [Werner Zwingman]_{PER} said on Wednesday consumers should…

• HMM Model:
  – States $Y = \{\text{NA, BL, CL, BO, CO, BP, CP}\}$ represent beginnings (BL, BO, BP) and continuations (CL, CO, CP) of chunks, as well as other words (NA)
  – Observations $X = V$ are words
  – Transition dist’n $q(y_i|y_{i-1})$ models the tag sequences
  – Emission dist’n $e(x_i|y_i)$ models words given their type
Low Frequency Words: An Example

• Named Entity Recognition [Bickel et. al, 1999]
  – Used the following word classes for infrequent words:

<table>
<thead>
<tr>
<th>Word class</th>
<th>Example</th>
<th>Intuition</th>
</tr>
</thead>
<tbody>
<tr>
<td>twoDigitNum</td>
<td>90</td>
<td>Two digit year</td>
</tr>
<tr>
<td>fourDigitNum</td>
<td>1990</td>
<td>Four digit year</td>
</tr>
<tr>
<td>containsDigitAndAlpha</td>
<td>A8956-67</td>
<td>Product code</td>
</tr>
<tr>
<td>containsDigitAndDash</td>
<td>09-96</td>
<td>Date</td>
</tr>
<tr>
<td>containsDigitAndSlash</td>
<td>11/9/89</td>
<td>Date</td>
</tr>
<tr>
<td>containsDigitAndComma</td>
<td>23,000.00</td>
<td>Monetary amount</td>
</tr>
<tr>
<td>containsDigitAndPeriod</td>
<td>1.00</td>
<td>Monetary amount, percentage</td>
</tr>
<tr>
<td>othernum</td>
<td>456789</td>
<td>Other number</td>
</tr>
<tr>
<td>allCaps</td>
<td>BBN</td>
<td>Organization</td>
</tr>
<tr>
<td>capPeriod</td>
<td>M.</td>
<td>Person name initial</td>
</tr>
<tr>
<td>firstWord</td>
<td>first word of sentence</td>
<td>no useful capitalization information</td>
</tr>
<tr>
<td>initCap</td>
<td>Sally</td>
<td>Capitalized word</td>
</tr>
<tr>
<td>lowercase</td>
<td>can</td>
<td>Uncapitalized word</td>
</tr>
<tr>
<td>other</td>
<td>,</td>
<td>Punctuation marks, all other words</td>
</tr>
</tbody>
</table>
Low Frequency Words: An Example

Profits/NA soared/NA at/NA Boeing/SO Co./CO ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA first/NA quarter/NA results/NA ./NA

• NA = No entity
• SO = Start Organization
• CO = Continue Organization
• SL = Start Location
• CL = Continue Location
• …
Low Frequency Words: An Example

Profits/NA soared/NA at/NA Boeing/SO Co./CO ,/NA easily/NA topping/NA forecasts/NA on/NA Wall/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP Mulally/CP announced/NA first/NA quarter/NA results/NA ./NA

firstword/NA soared/NA at/NA initCap/SC Co./CC ,/NA easily/NA lowercase/NA forecasts/NA on/NA initCap/SL Street/CL ,/NA as/NA their/NA CEO/NA Alan/SP initCap/CP announced/NA first/NA quarter/NA results/NA ./NA

- NA = No entity
- SO = Start Organization
- CO = Continue Organization
- SL = Start Location
- CL = Continue Location
- ...
HMM Inference and Learning

• Learning
  – Maximum likelihood: transitions $q$ and emissions $e$
    $$p(x_1 \ldots x_n, y_1 \ldots y_n) = q(\text{STOP}|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1}) e(x_i|y_i)$$

• Inference
  – Viterbi
    $$y^* = \arg \max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n)$$
  – Forward backward
    $$p(x_1 \ldots x_n, y_i) = \sum_{y_1 \ldots y_i-1} \sum_{y_{i+1} \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n)$$
Inference (Decoding)

• **Problem:** find the most likely (Viterbi) sequence under the model

\[ y^* = \arg \max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n) \]

• Given model parameters, we can score any sequence pair

  NNP  VBZ  NN  NNS  CD  NN  
  Fed  raises  interest  rates  0.5  percent  

  q(NNP|  ) e(Fed|NNP) q(VBZ|NNP) e(raises|VBZ) q(NN|VBZ)…..

• In principle, we’re done – list all possible tag sequences, score each one, pick the best one (the Viterbi state sequence)

  NNP  VBZ  NN  NNS  CD  NN  \( \rightarrow \) \( \log p(x, y) = -23 \)
  NNP  NNS  NN  NNS  CD  NN  \( \rightarrow \) \( \log(x, y) = -29 \)
  NNP  VBZ  VB  NNS  CD  NN  \( \rightarrow \) \( \log p(x, y) = -27 \)

Any issue?
Finding the Best Trajectory

• Too many trajectories (state sequences) to list
• Option 1: Beam Search
  – A beam is a set of partial hypotheses
  – Start with just the single empty trajectory
  – At each derivation step:
    • Consider all continuations of previous hypotheses
    • Discard most, keep top k

• Beam search often works OK in practice, but …
  • … sometimes you want the optimal answer
  • … and there’s usually a better option than naïve beams
The State Lattice / Trellis

START       Fed           raises       interest       rates       STOP

q(N|^)  e(Fed|N)  q(V|N)  e(raises|V)  e(interest|V)  e(rates|J)  e(STOP|V)

q(V|V)  q(V|V)  q(V|V)  q(V|V)  q(V|V)  q(V|V)  q(V|V)
Scoring a Sequence

\[ y^* = \arg \max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n) \]

\[ p(x_1 \ldots x_n, y_1 \ldots y_n) = q(\text{STOP}|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1}) e(x_i|y_i) \]

- Define \( \pi(i, y_i) \) to be the max score of a sequence of length \( i \) ending in tag \( y_i \)

\[ \pi(i, y_i) = \max_{y_1 \ldots y_{i-1}} p(x_1 \ldots x_i, y_1 \ldots y_i) \]

\[ = \max_{y_{i-1}} e(x_i|y_i) q(y_i|y_{i-1}) \max_{y_1 \ldots y_{i-2}} p(x_1 \ldots x_{i-1}, y_1 \ldots y_{i-1}) \]

\[ = \max_{y_{i-1}} e(x_i|y_i) q(y_i|y_{i-1}) \pi(i - 1, y_{i-1}) \]

- We can now design an efficient algorithm.
  - How?
The Viterbi Algorithm

Dynamic program for computing (for all \(i\))

\[
\pi(i, y_i) = \max_{y_1 \ldots y_{i-1}} p(x_1 \ldots x_i, y_1 \ldots y_i)
\]

Iterative computation:

\[
\pi(0, y_0) = \begin{cases} 
1 & \text{if } y_0 == START \\
0 & \text{otherwise}
\end{cases}
\]

For \(i = 1 \ldots n:\)

// Store score

\[
\pi(i, y_i) = \max_{y_{i-1}} e(x_i|y_i) q(y_i|y_{i-1}) \pi(i - 1, y_{i-1})
\]

// Store back-pointer

\[
bp(i, y_i) = \arg \max_{y_{i-1}} e(x_i|y_i) q(y_i|y_{i-1}) \pi(i - 1, y_{i-1})
\]
The State Lattice / Trellis

Tie breaking:
Prefer first

START
Fed
raises
interest
STOP

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<tr>
<th>from \ to</th>
<th>^</th>
<th>N</th>
<th>V</th>
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The State Lattice / Trellis

^ \pi = 1 
bp = null

\pi = 0 
bp = null

\pi = 0 
bp = null

\pi = 0 
bp = null

\pi = 0 
bp = N

\pi = 0 
bp = V

\pi = V
bp = N

\pi = 0 
bp = V

\pi = 0 
bp = V

\pi = 0 
bp = V

\pi = 0 
bp = V

\pi = 0 
bp = V

\pi = 0 
bp = V

\pi = 0.0206 
bp = V

\pi = 0.001512 
bp = V

\pi = 0.000831 
bp = V

\pi = 0.00040824 
bp = V

START
Fed
raises
interest
STOP

emissions
START
Fed
raises
interest
STOP

From \ to
\nN
V
$

\pi = 1
\pi = 0
\pi = 0
\pi = 0
\pi = 0
\pi = 0
\pi = 0
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\pi = 0
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bp = null
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bp = null
bp = null
bp = N
bp = V
bp = N
bp = V
bp = V
bp = V
bp = V
bp = V
bp = V
bp = V
bp = V

Tie breaking: Prefer first
The Viterbi Algorithm: Runtime

- In term of sentence length $n$?
  - Linear
- In term of number of states $|K|$?
  - Polynomial
  
  \[ \pi(i, y_i) = \max_{y_{i-1}} e(x_i | y_i) q(y_i | y_{i-1}) \pi(i - 1, y_{i-1}) \]

- Specifically:
  - $O(n|K|)$ entries in $\pi(i, y_i)$
  - $O(|K|)$ time to compute each $\pi(i, y_i)$

- Total runtime: $O(n|K|^2)$

- Q: Is this a practical algorithm?
- A: depends on $|K|$....
Tagsets in Different Languages

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<th>Language</th>
<th>Source</th>
<th># Tags</th>
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\[294^2 = 86436\]
\[45^2 = 2045\]
\[11^2 = 121\]

[Petrov et al. 2012]
HMM Inference and Learning

• Learning
  – Maximum likelihood: transitions $q$ and emissions $e$
    \[ p(x_1 \ldots x_n, y_1 \ldots y_n) = q(STOP|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1})e(x_i|y_i) \]

• Inference
  – Viterbi
    \[ y^* = \arg \max_{y_1 \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n) \]
  – Forward backward
    \[ p(x_1 \ldots x_n, y_i) = \sum_{y_1 \ldots y_{i-1}} \sum_{y_{i+1} \ldots y_n} p(x_1 \ldots x_n, y_1 \ldots y_n) \]
What about n-gram Taggers?

- States encode what is relevant about the past
- Transitions $P(s_i | s_{i-1})$ encode well-formed tag sequences
  - In a bigram tagger, states = tags
    $$s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \cdots \rightarrow s_n$$
    $$x_1 \rightarrow x_2 \rightarrow \cdots \rightarrow x_n$$
  - In a trigram tagger, states = tag pairs
    $$s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \cdots \rightarrow s_n$$
    $$x_1 \rightarrow x_2 \rightarrow \cdots \rightarrow x_n$$
The State Lattice / Trellis

Not all edges are allowed

\[
\begin{align*}
q(N|\text{^,^}) & \quad e(\text{Fed}|N) & q(D|\text{^,V}) & \quad e(\text{raises}|D) & q(V|N,D) & \quad e(\text{interest}|V) \\
\text{START} & \quad ^ & \quad N & \quad \text{raises} & \quad D & \quad \text{interest} & \quad V & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} \\
\text{Fed} & \quad N & \quad \text{raises} & \quad D & \quad \text{interest} & \quad V & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} \\
\text{interest} & \quad V & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} & \quad \text{interest} \\
\$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ & \quad \$ \\
\end{align*}
\]
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- $294^2 = 86436$
- $294^4 = 7471182096$
- $45^2 = 2045$
- $45^4 = 4100625$
- $11^2 = 121$
- $11^4 = 14641$

[Source: Petrov et al. 2012]
Some Numbers

• Rough accuracies:
  – Most freq tag:
  – Trigram HMM:
  – TnT (Brants, 2000):
    • A carefully smoothed trigram tagger
    • Suffix trees for emissions
  – Upper bound: ~98%
Re-visit $P(x \mid y)$

• Reality check:
  – What if we drop the sequence?
    • Use only $P(x \mid y)$
  – Most frequent tag:
    • 90.3% with a so-so unknown word model
  – Can we do better?
What about better features?

• Looking at a word and its environment
  – Add in previous / next word the __
  – Previous / next word shapes X __ X
  – Occurrence pattern features [X: x X occurs]
  – Crude entity detection __ ..... (Inc.|Co.)
  – Phrasal verb in sentence? put ...... __
  – Conjunctions of these things

• Uses lots of features: > 200K
Some Numbers

• Rough accuracies:
  – Most freq tag: ~90% / ~50%
  – Trigram HMM: ~95% / ~55%
  – TnT (Brants, 2000): 96.7% / 85.5%
  – MaxEnt P(y | x)

• What does this tell us about sequence models?
• How do we add more features to our sequence models?

  – Upper bound: ~98%
MEMM Taggers

One step up: also condition on previous tags:

\[
p(y_1 \ldots y_n | x_1 \ldots x_n) = \prod_{i=1}^{n} p(y_i | y_1 \ldots y_{i-1}, x_1 \ldots x_n)
\]

= \prod_{i=1}^{n} p(y_i | y_{i-1}, x_1 \ldots x_n)

• Training:
  – Train \( p(y_i | y_{i-1}, x_1 \ldots x_n) \) as a discrete log-linear (MaxEnt) model
• Scoring:

\[
p(y_i | y_{i-1}, x_1 \ldots x_n) = \frac{e^{w \cdot \phi(x_1 \ldots x_n, i, y_{i-1}, y_i)}}{\sum_{y'} e^{w \cdot \phi(x_1 \ldots x_n, i, y_{i-1}, y')}}
\]

• This is referred to as an MEMM tagger [Ratnaparkhi 96]
HMM vs. MEMM

• HMM models joint distribution:

\[ p(x_1 \ldots x_n, y_1 \ldots y_n) = q(\text{STOP}|y_n) \prod_{i=1}^{n} q(y_i|y_{i-1}) e(x_i|y_i) \]

• MEMM models conditioned distribution:

\[ p(y_1 \ldots y_n|x_1 \ldots x_n) = \prod_{i=1}^{n} p(y_i|y_1 \ldots y_{i-1}, x_1 \ldots x_n) \]
Decoding MEMM Taggers

- Scoring:

\[
p(y_i|y_{i-1}, x_1 \ldots x_n) = \frac{e^{w \cdot \phi(x_1 \ldots x_n, i, y_{i-1}, y_i)}}{\sum_{y'} e^{w \cdot \phi(x_1 \ldots x_n, i, y_{i-1}, y')}}
\]

- Beam search is effective – why?
- Guarantees? Optimal?
- Can we do better?
START       Fed           raises       interest       rates       STOP

\[
\begin{align*}
q(N|\wedge) & \quad e(Fed|N) \\
q(V|N) & \quad e(raises|V) \quad e(interest|V) \\
q(V|V) & \quad e(rates|J) \quad e(STOP|V)
\end{align*}
\]
The MEMM State Lattice / Trellis

START       Fed           raises       interest       rates       STOP
^                N               V              V                   J               V
q(N|^, X)        q(V|N, X)       q(V|V, X)      q(V|V, X)      q(J|V, X)      q(V|J, X)
$               $               $              $                   $               $
Decoding MEMM Taggers

• Decoding MaxEnt taggers:
  – Just like decoding HMMs
  – Viterbi, beam search

• Viterbi algorithm (HMMs):
  – Define $\pi(i, y_i)$ to be the max score of a sequence of length $i$ ending in tag $y_i$

$$\pi(i, y_i) = \max_{y_{i-1}} e(x_i | y_i) q(y_i | y_{i-1}) \pi(i - 1, y_{i-1})$$

• Viterbi algorithm (MaxEnt):
  – Can use same algorithm for MEMMs, just need to redefine $\pi(i, y_i)$!

$$\pi(i, y_i) = \max_{y_{i-1}} p(y_i | y_{i-1}, x_1 \ldots x_m) \pi(i - 1, y_{i-1})$$
Some Numbers

- Rough accuracies:
  - Most freq tag: \(~90\% / \sim50\%\)
  - Trigram HMM: \(~95\% / \sim55\%\)
  - TnT (Brants, 2000): \(96.7\% / 85.5\%\)
  - MaxEnt \(P(y \mid x)\): \(93.7\% / 82.6\%\)
  - MEMM tagger 1:
  - Upper bound: \(~98\%\)

[Ratnaparkhi 1996]
# Feature Development

Common errors:

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**NN/JJ** NN **NN** VBD RP/IN DT NN **RB** VBD/VBN NNS

official knowledge made up the story recently sold shares

[Toutanova and Manning 2000]
Some Numbers

• Rough accuracies:
  – Most freq tag: ~90% / ~50%
  – Trigram HMM: ~95% / ~55%
  – TnT (Brants, 2000): 96.7% / 85.5%
  – MaxEnt P(y | x) 93.7% / 82.6%
  – MEMM tagger 1: 96.7% / 84.5%
  – MEMM tagger 2: 96.7% / 84.5%

  – Upper bound: ~98%

[Toutanova and Manning 2000]
Locally Normalized Models

• So far:
  – Probabilities are product of \textit{locally normalized} probabilities
  – Is this bad?

• \textbf{Label bias}
  – States with fewer transitions are likely to be preferred because normalization is local
  – Extreme case: What happens if there is only one outgoing arc? Does it matter what the observation is?
Locally Normalized Models

• So far:
  – Probabilities are product of locally normalized probabilities
  – Is this bad?

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</tbody>
</table>

AAA →
ABB →

B → B transitions are likely to take over even if rarely observed!
Global Discriminative Taggers

• Discriminative sequence models
  – CRFs (also Perceptrons)
  – Do not decompose training into independent local regions
  – Can be very slow* to train – require repeated inference on training set

* Relatively slow. NN models are much slower.
Linear Models: Perceptron

- The perceptron algorithm
  - Iteratively processes the data, reacting to training errors
  - Can be thought of as trying to drive down training error

- The (online structured) perceptron algorithm:
  - Start with zero weights
  - Visit training instances \((X^{(i)}, Y^{(i)})\) one by one
    - Make a prediction
      \[
      Y^* = \arg \max_Y w \cdot \phi(X^{(i)}, Y)
      \]
    - If correct \((Y^* = Y^{(i)})\):
      - no change, goto next example!
    - If wrong:
      - adjust weights: 
        \[
        w = w + \phi(X^{(i)}, Y^{(i)}) - \phi(X^{(i)}, y^*)
        \]

- **Challenge**: How to compute argmax efficiently?
Decoding

- **Linear Perceptron**
  \[ Y^* = \arg \max_Y w \cdot \phi(X, Y) \]
  - Features must be local, for \( X = x_1 \ldots x_n \), and \( Y = y_1 \ldots y_m \)
  \[ \phi(X, Y) = \sum_{j=1}^{n} \phi(X, j, y_{j-1}, y_j) \]
The MEMM State Lattice / Trellis

START       Fed           raises       interest         rates         STOP
^                N               V              V                   J               V

q(N|^)  q(V|N)  q(V|V)  q(J|V)  q(V|J)
The Perceptron State Lattice / Trellis

START       Fed           raises       interest         rates         STOP
^                N               V              V                   J               V

w•Φ(X,1,N,∧,X)  w•Φ(X,2,V,N)  w•Φ(X,3,V,V)  w•Φ(X,4,J,V)  w•Φ(X,5,V,J)
Decoding

• Linear Perceptron \( Y^* = \arg \max_Y w \cdot \phi(X, Y) \)
  
  – Features must be local, for \( X = x_1 \ldots x_n \), and \( Y = y_1 \ldots y_n \)
    \[
    \phi(X, Y) = \sum_{j=1}^{n} \phi(X, j, y_{j-1}, y_j)
    \]
  
  – Define \( \pi(i, y_i) \) to be the max score of a sequence of length \( i \)
    ending in tag \( y_i \)
    \[
    \pi(i, y_i) = \max_{y_{i-1}} w \cdot \phi(X, i, y_{i-1}, y_i) + \pi(i - 1, y_{i-1})
    \]

• Viterbi algorithm (HMMs):
  \[
  \pi(i, y_i) = \max_{y_{i-1}} e(x_i | y_i) q(y_i | y_{i-1}) \pi(i - 1, y_{i-1})
  \]

• Viterbi algorithm (Maxent):
  \[
  \pi(i, y_i) = \max_{y_{i-1}} p(y_i | y_{i-1}, x_1 \ldots x_m) \pi(i - 1, y_{i-1})
  \]
Some Numbers

• Rough accuracies:
  – Most freq tag: ~90% / ~50%
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  – MEMM tagger 2: 96.8% / 86.9%
  – Perceptron:

  – Upper bound: ~98%

[Collins 2002]
Conditional Random Fields (CRFs)

• What did we lose with the Perceptron?
  – No probabilities
  – Let’s try again with a probabilistic model
CRFs

- Maximum entropy (logistic regression)

Sentence: $X = x_1 \ldots x_n$

$$p(Y \mid X; w) = \frac{1}{Z(X; w)} \exp(w \cdot \phi(X, Y))$$

Tag Sequence: $Y = y_1 \ldots y_n$

- Learning: maximize the (log) conditional likelihood of training data
  $$\{(X^{(i)}, Y^{(i)})\}_{i=1}^m$$

- Computational challenges?
  - Most likely tag sequence, normalization constant, gradient

\[\frac{\partial}{\partial w_j} L(w) = \sum_{i=1}^m \left( \phi_j(X^{(i)}, Y^{(i)}) - \sum_Y p(Y \mid X^{(i)}; w) \phi_j(X^{(i)}, Y) \right) - \lambda w_j\]

[Lafferty et al. 2001]
Decoding

- CRFs
  - Features must be local, for \( x = x_1 \ldots x_n \), and \( y = y_1 \ldots y_n \)

\[
p(Y \mid X; w) = \frac{\exp(w \cdot \phi(X, Y))}{\sum_{Y'} \exp(w \cdot \phi(X, Y'))} \quad \phi(X, Y) = \sum_{j=1}^{n} \phi(X, j, y_{j-1}, y_j)
\]

\[
\arg \max_Y \frac{\exp(w \cdot \phi(X, Y))}{\sum_{Y'} \exp(w \cdot \phi(X, Y'))} = \arg \max_Y \exp(w \cdot \phi(X, Y)) = \arg \max_Y w \cdot \phi(X, Y)
\]

- Looks familiar?
- Same as linear Perceptron!

\[
\pi(i, y_i) = \max_{y_{i-1}} \phi(x, i, y_{i-1}, y_i) + \pi(i - 1, y_{i-1})
\]
CRFs: Computing Normalization

\[ p(Y \mid X; w) = \frac{\exp(w \cdot \phi(X, Y))}{\sum_{Y'} \exp(w \cdot \phi(X, Y'))} \]

\[ \phi(X, Y) = \sum_{j=1}^{n} \phi(X, j, y_{j-1}, y_j) \]

\[ \sum_{Y'} \exp(w \cdot \phi(X, Y')) = \sum_{Y'} \exp \left( \sum_{j=1}^{n} w \cdot \phi(X, j, y_{j-1}, y_j) \right) \]

\[ = \sum_{Y'} \prod_{j=1}^{n} \exp(w \cdot \phi(X, j, y_{j-1}, y_j)) \]

Define \( \text{norm}(i, y_i) \) to sum of scores for sequences ending in position \( i \)

\[ \text{norm}(i, y_i) = \sum_{y_{i-1}} \exp(w \cdot \phi(X, i, y_{i-1}, y_i)) \text{norm}(i - 1, y_{i-1}) \]

- Forward algorithm! Remember HMM case:

\[ \pi(i, y_i) = \max_{y_{i-1}} e(x_i \mid y_i) q(y_i \mid y_{i-1}) \pi(i - 1, y_{i-1}) \]
CRFs: Computing Gradient

\[ p(Y \mid X; w) = \frac{\exp(w \cdot \phi(X, Y))}{\sum_{Y', \exp(w \cdot \phi(X, Y'))} \phi(X, Y)} \]

\[ \phi(X, Y) = \sum_{j=1}^{n} \phi(X, j, y_{j-1}, y_{j}) \]

\[ \frac{\partial}{\partial w_{j}} L(w) = \sum_{i=1}^{m} \left( \phi_{j}(X^{(i)}, Y^{(i)}) - \sum_{Y} p(Y \mid X^{(i)}; w) \phi_{j}(X^{(i)}, Y) \right) - \lambda w_{j} \]

\[ \sum_{Y} p(Y \mid X^{(i)}; w) \phi_{j}(X^{(i)}, Y) = \sum_{Y} p(Y \mid X^{(i)}; w) \sum_{k=1}^{n} \phi_{j}(X^{(i)}, k, y_{k-1}, y_{k}) \]

\[ = \sum_{k=1}^{n} \sum_{a,b} \sum_{y_{k-1}=a, y_{k}=b} p(Y \mid X^{(i)}; w) \phi_{j}(X^{(i)}, k, y_{k-1}, y_{k}) \]

- Can compute with the Forward Backward algorithm

See notes for full details!
Some Numbers

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  – Perceptron: 97.1%
  – CRF++:
  – Upper bound: ~98%

[Sun 2014]
Cyclic Network

- Train two MEMMs, combine scores
- And be very careful
  - Tune regularization
  - Try lots of different features
  - See paper for full details

(a) Left-to-Right CMM
(b) Right-to-Left CMM
(c) Bidirectional Dependency Network

[Toutanova et al. 2003]
Some Numbers

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  – MEMM tagger 1: 96.7% / 84.5%
  – MEMM tagger 2: 96.8% / 86.9%
  – Perceptron: 97.1%
  – CRF++: 97.3%
  – Cyclic tagger: 
  – Upper bound: ~98%

[Toutanova et al. 2003]
Summary

- For tagging, the change from generative to **discriminative** model does not by itself result in great improvement.
- One profits from models for specifying dependence on **overlapping features** of the observation such as spelling, suffix analysis, etc.
- MEMMs allow **integration of rich features** of the observations.
- This **additional power** (of the MEMM, CRF, Perceptron models) has been shown to result in improvements in accuracy.
- The higher accuracy of discriminative models comes at the price of **much slower training**.
Domain Effects

• Accuracies degrade outside of domain
  – Up to triple error rate
  – Usually make the most errors on the things you care about in the domain (e.g. protein names)

• Open questions
  – How to effectively exploit unlabeled data from a new domain (what could we gain?)
  – How to best incorporate domain lexica in a principled way (e.g. UMLS specialist lexicon, ontologies)