Quiz 1 (on Canvas)

Closed book / closed note
Starts at 1:25pm
Ends at 1:33pm (8 minutes)
CS5670: Computer Vision
Image Resampling & Interpolation
Announcements

• Project 1 released, due Friday, February 9 by 8pm on GitHub Classroom
  – Project to be done solo (teams of one)
  – Artifact due Monday, Feb 12 by 8pm
Reading

• Szeliski 2.3.1, 3.4-3.5
Image scaling

This image is too big to fit on the screen. How can we generate a half-sized version?
Image sub-sampling

Throw away every other row and column to create a 1/2 size image - called *image sub-sampling*

Source: S. Seitz
Image sub-sampling

Why does this look so crufty?

Source: S. Seitz
Image sub-sampling – another example

Source: F. Durand
Even worse for synthetic images

Source: L. Zhang
Aliasing

• Occurs when your sampling rate is not high enough to capture the amount of detail in your image
• Can give you the wrong signal/image—an alias

• To do sampling right, need to understand the structure of your signal/image
• Enter Monsieur Fourier…
  – “But what is the Fourier Transform? A visual introduction.”
    https://www.youtube.com/watch?v=spUNpyF58BY
• To avoid aliasing:
  – sampling rate $\geq 2 \times$ max frequency in the image
    • said another way: $\geq$ two samples per cycle
  – This minimum sampling rate is called the Nyquist rate

Source: L. Zhang
Wagon-wheel effect

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

(See http://www.michaelbach.de/ot/mot-wagonWheel/index.html)
Wagon-wheel effect

https://en.wikipedia.org/wiki/Wagon-wheel_effect
Temporal aliasing – helicopter blades

https://www.youtube.com/watch?v=yr3ngmRuGUC
Aliasing in practice
Nyquist limit – 2D example

Good sampling

Bad sampling
Aliasing

• When downsampling by a factor of two
  – Original image has frequencies that are too high

• How can we fix this?
Gaussian pre-filtering

- Solution: filter the image, \textit{then} subsample

Source: S. Seitz
Subsampling with Gaussian pre-filtering

- Solution: filter the image, *then* subsample

Source: S. Seitz
Compare with...

1/2

1/4 (2x zoom)

1/8 (4x zoom)

Source: S. Seitz
Gaussian pre-filtering

- Solution: filter the image, then subsample
Gaussian pyramid

\[
\begin{align*}
  F_0 & \xrightarrow{\text{blur}} \quad \text{subsample} & \quad F_1 & \xrightarrow{\text{blur}} \quad \text{subsample} \\
  F_0^* & \quad H & \quad F_1^* & \quad H
\end{align*}
\]
**Gaussian pyramids** [Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

Gaussian Pyramids have all sorts of applications in computer vision

Source: S. Seitz
Gaussian pyramids [Burt and Adelson, 1983]

Idea: Represent N×N image as a “pyramid” of 1×1, 2×2, 4×4, ..., 2^k×2^k images (assuming N=2^k)

- How much space does a Gaussian pyramid take compared to the original image?

Source: S. Seitz
Gaussian pyramid
How much space (number of pixels) does a Gaussian pyramid of an image take compared to the original image?
Gaussian pyramid

Answer: 4/3 the amount of space as the original image alone
See https://en.wikipedia.org/wiki/Geometric_series
Back to the checkerboard

• What should happen when you make the checkerboard smaller and smaller?

Naive subsampling  Proper prefiltering ("antialiasing")

Image turns grey! (Average of black and white squares, because each pixel contains both.)
Questions?
Upsampling

• This image is too small for this screen:
• How can we make it 10 times as big?
• Simplest approach:
  - repeat each row
  - and column 10 times
• (“Nearest neighbor interpolation”)
Image interpolation

Recall that a digital image is formed as follows:

\[ F[x, y] = \text{quantize}\{f(xd, yd)\} \]

- It is a discrete point-sampling of a continuous function.
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale.

Adapted from: S. Seitz
Image interpolation

Recall that a digital image is formed as follows:

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Adapted from: S. Seitz
Image interpolation

- What if we don’t know $f$?
  - Guess an approximation: $\tilde{f}$
  - Can be done in a principled way: filtering
  - Convert $F$ to a continuous function:
    $$f_F(x) = F\left(\frac{x}{d}\right) \text{ when } \frac{x}{d} \text{ is an integer, 0 otherwise}$$
  - Reconstrcut by convolution with a reconstruction filter, $h$
    $$\tilde{f} = h * f_F$$

Adapted from: S. Seitz
Image interpolation

- **sinc(x)**
  - “Ideal” reconstruction

- **II(x)**
  - Nearest-neighbor interpolation

- **Λ(x)**
  - Linear interpolation

- **gauss(x)**
  - Gaussian reconstruction

Source: B. Curless
Reconstruction filters

- What does the 2D version of this hat function look like?

  \[ h(x) \]
  \[ h(x, y) \]
  performs linear interpolation
  (tent function) performs \textbf{bilinear interpolation}

Often implemented without cross-correlation

- E.g., \url{http://en.wikipedia.org/wiki/Bilinear_interpolation}

Better filters give better resampled images

- \textbf{Bicubic} is common choice

\[
r(x) = \begin{cases} 
  \frac{(12 - 98 - 6C)|x| + (-18 + 128 + 6C)|x|^2 + (6 - 2B)}{6} & \text{for } |x| < 1 \\
  \frac{(1 - 8 - 6C)|x|^2 + (68 + 30C)|x|^3 + (-128 - 48C)|x|^4 + (88 + 24C)|x|^5}{6} & \text{for } 1 \leq |x| < 2 \\
  0 & \text{otherwise}
\end{cases}
\]

Cubic reconstruction filter
Image interpolation

Original image: ð® x 10

Nearest-neighbor interpolation  Bilinear interpolation  Bicubic interpolation
Image interpolation

Also used for resampling
Raster-to-vector graphics

Vector Magic

Simply the Best Auto-Tracer in the World
Depixelating Pixel Art
Modern methods

From Romano, et al: RAISR: Rapid and Accurate Image Super Resolution, 
https://arxiv.org/abs/1606.01299
Super-resolution with multiple images

• Can do better upsampling if you have multiple images of the scene taken with small (subpixel) shifts
• Some cellphone cameras (like the Google Pixel line) capture a burst of photos
• Can we use that burst for upsampling?
Google Pixel 3 Super Res Zoom

Effect of hand tremor as seen in a cropped burst of photos, after global alignment

Example photo with and without super res zoom (smart burst align and merge)

Summary

• Key points:
  – **Subsampling an image** can cause aliasing. Better is to blur (“pre-filter”) to remote high frequencies then downsample
  – If you repeatedly blur and downsample by 2x, you get a Gaussian pyramid
  – **Upsampling an image** requires interpolation. This can be posed as convolution with a “reconstruction kernel”
Questions?