Image Resampling & Interpolation
This image is too big to fit on the screen. How can we generate a half-sized version?
Throw away every other row and column to create a 1/2 size image - called *image sub-sampling*.

Source: S. Seitz
Why does this look so crufty?

Source: S. Seitz
Image sub-sampling
Even worse for synthetic images

Source: L. Zhang
• Occurs when your sampling rate is not high enough to capture the amount of detail in your image

• Can give you the wrong signal/image—an alias

• To do sampling right, need to understand the structure of your signal/image

• Enter Monsieur Fourier...

• To avoid aliasing:
  – sampling rate $\geq 2 \times$ max frequency in the image
    • said another way: $\geq$ two samples per cycle
  – This minimum sampling rate is called the **Nyquist rate**
**Wagon-wheel effect**

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)

(See [http://www.michaelbach.de/ot/mot_wagonWheel/index.html](http://www.michaelbach.de/ot/mot_wagonWheel/index.html))

Source: L. Zhang
Nyquist limit – 2D example

Good sampling

Bad sampling
Aliasing

• When downsampling by a factor of two
  – Original image has frequencies that are too high

• How can we fix this?
Gaussian pre-filtering

- Solution: filter the image, *then* subsample
Subsampling with Gaussian pre-filtering

• Solution: filter the image, *then* subsample

Source: S. Seitz
Compare with...

1/2  1/4  (2x zoom)  1/8  (4x zoom)

Source: S. Seitz
Gaussian pre-filtering

- Solution: filter the image, *then* subsample
Gaussian pyramid
Gaussian pyramids
[Burt and Adelson, 1983]

- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

Gaussian Pyramids have all sorts of applications in computer vision

Source: S. Seitz
Gaussian pyramids
[Burt and Adelson, 1983]

• How much space does a Gaussian pyramid take compared to the original image?

Source: S. Seitz
Gaussian Pyramid
Questions?
Upsampling

• This image is too small for this screen:

• How can we make it 10 times as big?

• Simplest approach:
  repeat each row
  and column 10 times

• (“Nearest neighbor interpolation”)
Image interpolation

Recall how a digital image is formed

\[ F[x, y] = \text{quantize}\{f(xd, yd)\} \]

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

Adapted from: S. Seitz
Image interpolation

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Adapted from: S. Seitz
Image interpolation

- What if we don’t know \( f \) ?
  - Guess an approximation: \( \tilde{f} \)
  - Can be done in a principled way: filtering
  - Convert \( F \) to a continuous function:
    \[
    f_F(x) = F\left(\frac{x}{d}\right) \text{ when } \frac{x}{d} \text{ is an integer, } 0 \text{ otherwise}
    \]
  - Reconstruct by convolution with a reconstruction filter, \( h \)
    \[
    \tilde{f} = h \ast f_F
    \]

Adapted from: S. Seitz
Image interpolation

- $\text{sinc}(x)$: "Ideal" reconstruction
- $\text{II}(x)$: Nearest-neighbor interpolation
- $\text{\Lambda}(x)$: Linear interpolation
- $\text{gauss}(x)$: Gaussian reconstruction

Source: B. Curless
Reconstruction filters

• What does the 2D version of this hat function look like?

\[ h(x) \]

performs linear interpolation

\[ h(x, y) \]

(tent function) performs bilinear interpolation

Often implemented without cross-correlation

• E.g., http://en.wikipedia.org/wiki/Bilinear_interpolation

Better filters give better resampled images

• **Bicubic** is common choice

\[
r(x) = \begin{cases} 
\frac{1}{6} & \text{for } |x| < 1 \\
\frac{(12 - 98 - 6C)|x|^4 + (-18 + 12B + 6C)|x|^3 + (6 - 2B)}{(8B + 30C)|x|^2 + (-12B - 48C)|x| + (88 + 24C)} & \text{for } 1 \leq |x| < 2 \\
0 & \text{otherwise}
\end{cases}
\]
Image interpolation

Original image:  x 10

Nearest-neighbor interpolation  Bilinear interpolation  Bicubic interpolation
Image interpolation

Also used for *resampling*
Depixelating Pixel Art
Modern methods

Questions?