CS**5643**

08 Collision detection

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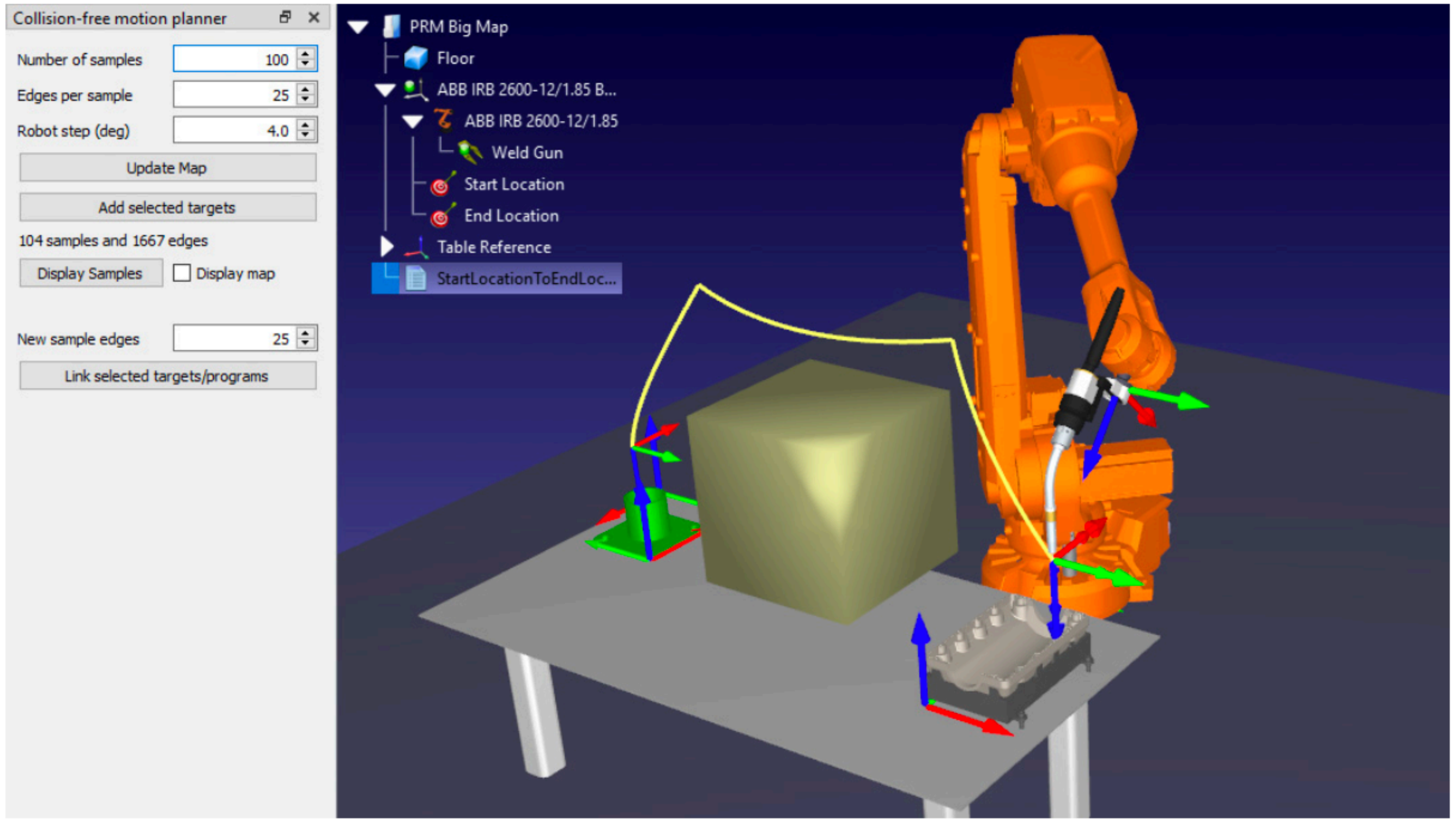
(many images borrowed from Doug James's Stanford CS 248b slides)

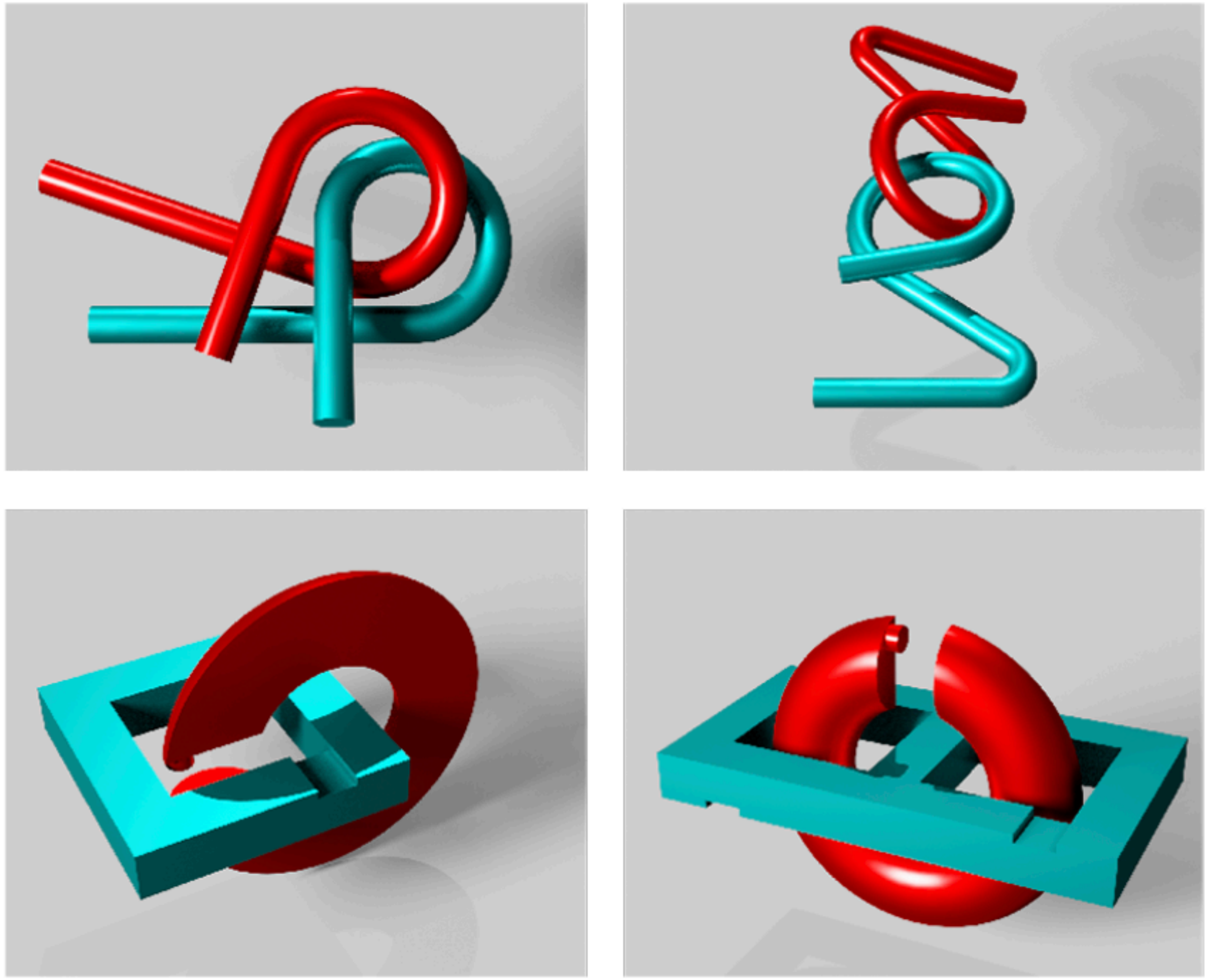
Collision detection

Goal: determine if two objects collide during a particular movement

- example: path planning for robotics or puzzles
- need to verify a particular motion path can execute with no collisions



https://robodk.com/blog/motion-planning-trend/



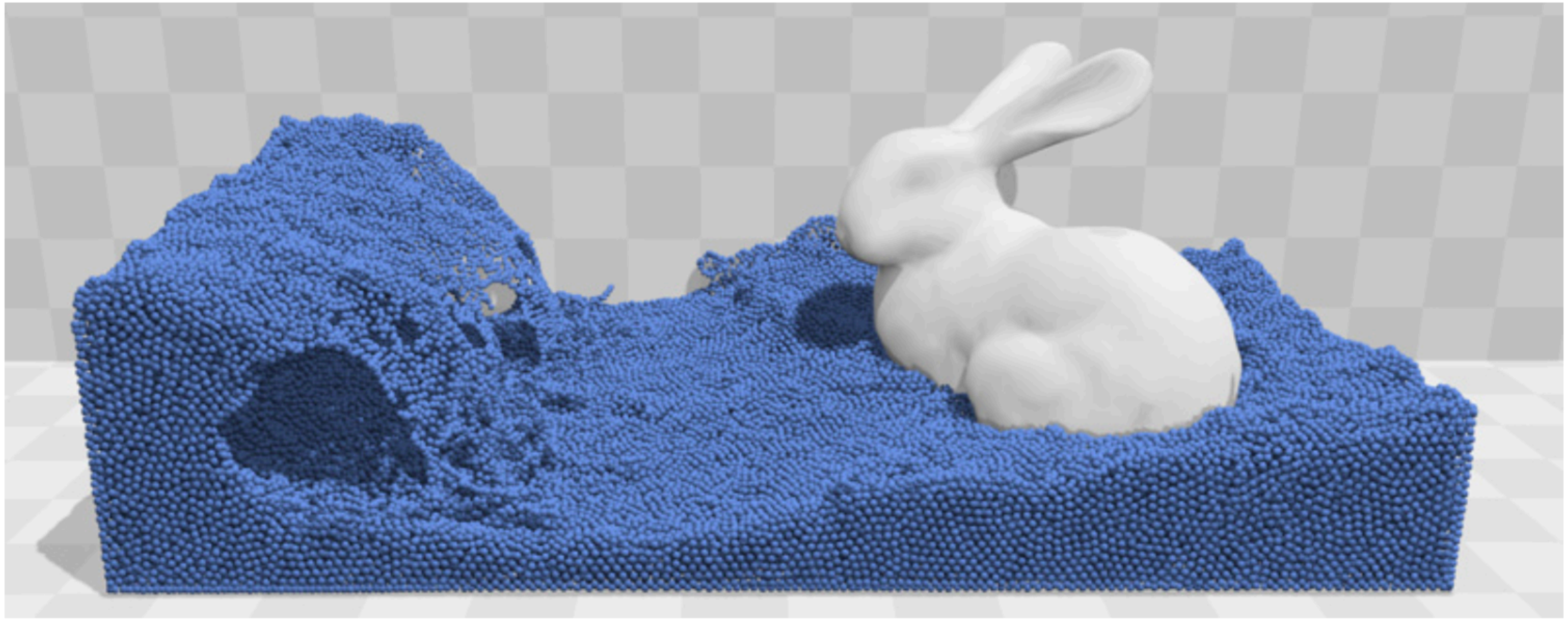
https://xinyazhang.gitlab.io/puzzletunneldiscovery/

slide borrowed from Doug James

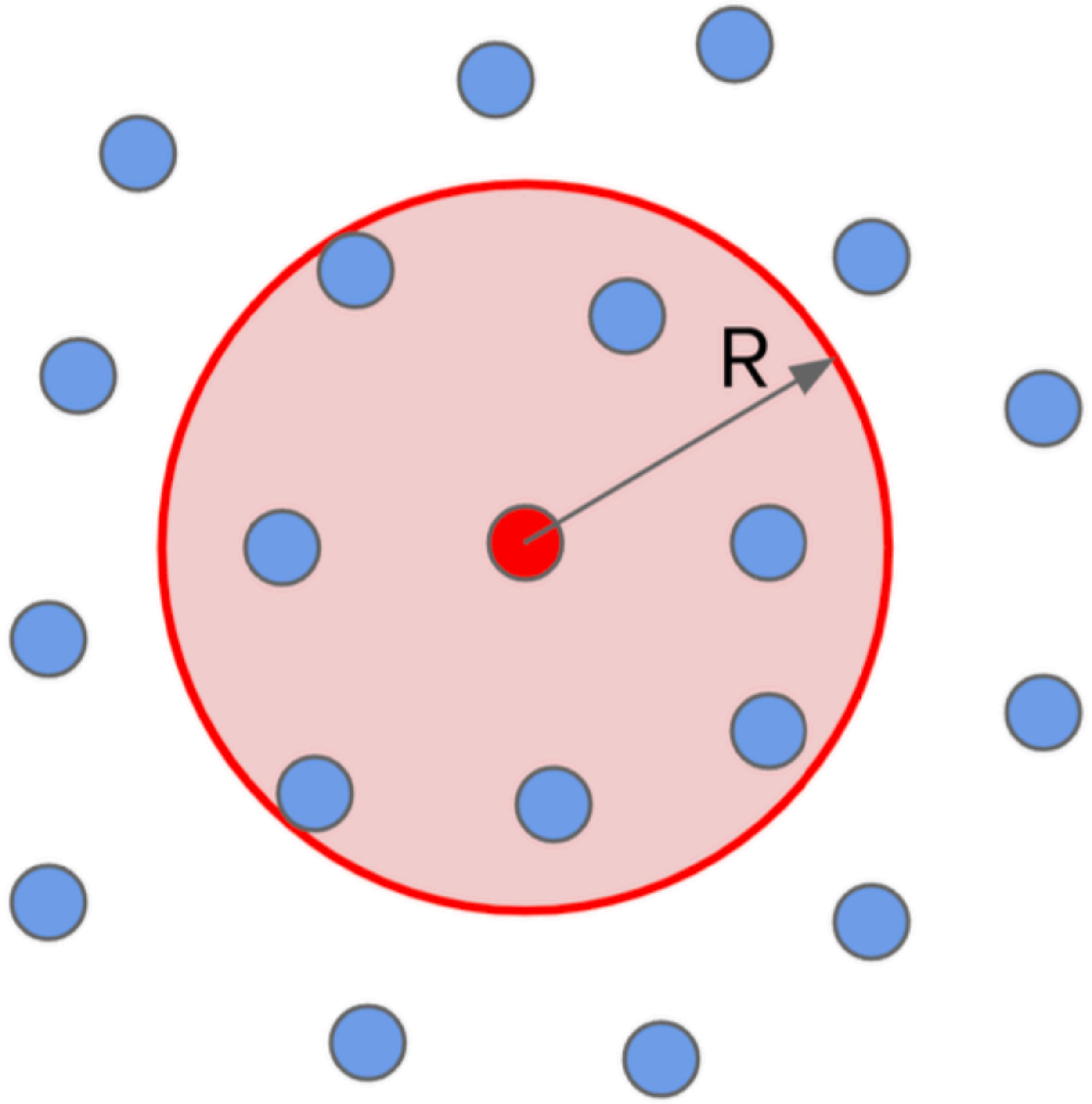
Proximity queries

Goal: detect when two objects approach within a threshold

- example: particle based fluid simulation
- each particle needs interacts with all particles closer than distance R



Position Based Fluids [Macklin and Mueller 2013]



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Continuous vs. instantaneous collision detection

Version 1: "Are these two objects colliding right now?"

- instantaneous collision detection
- can miss collisions if you check once per frame

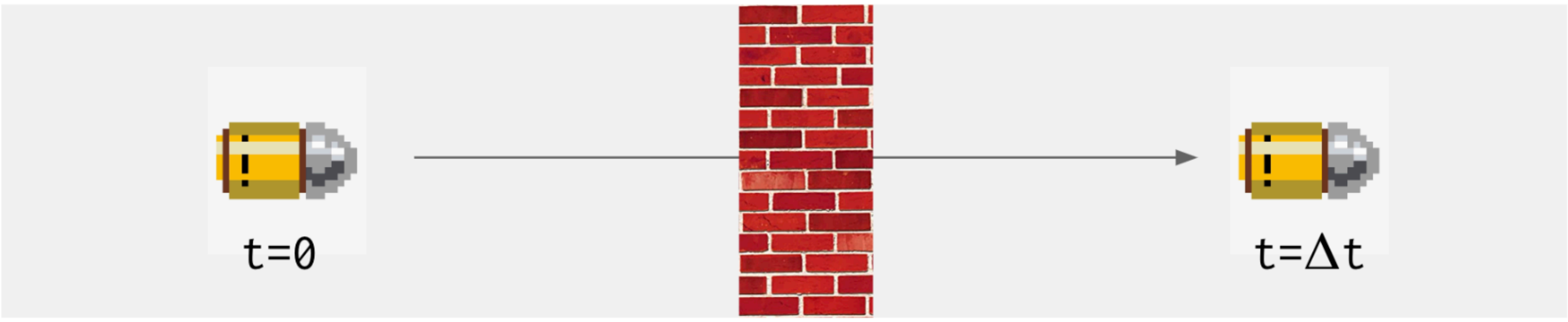
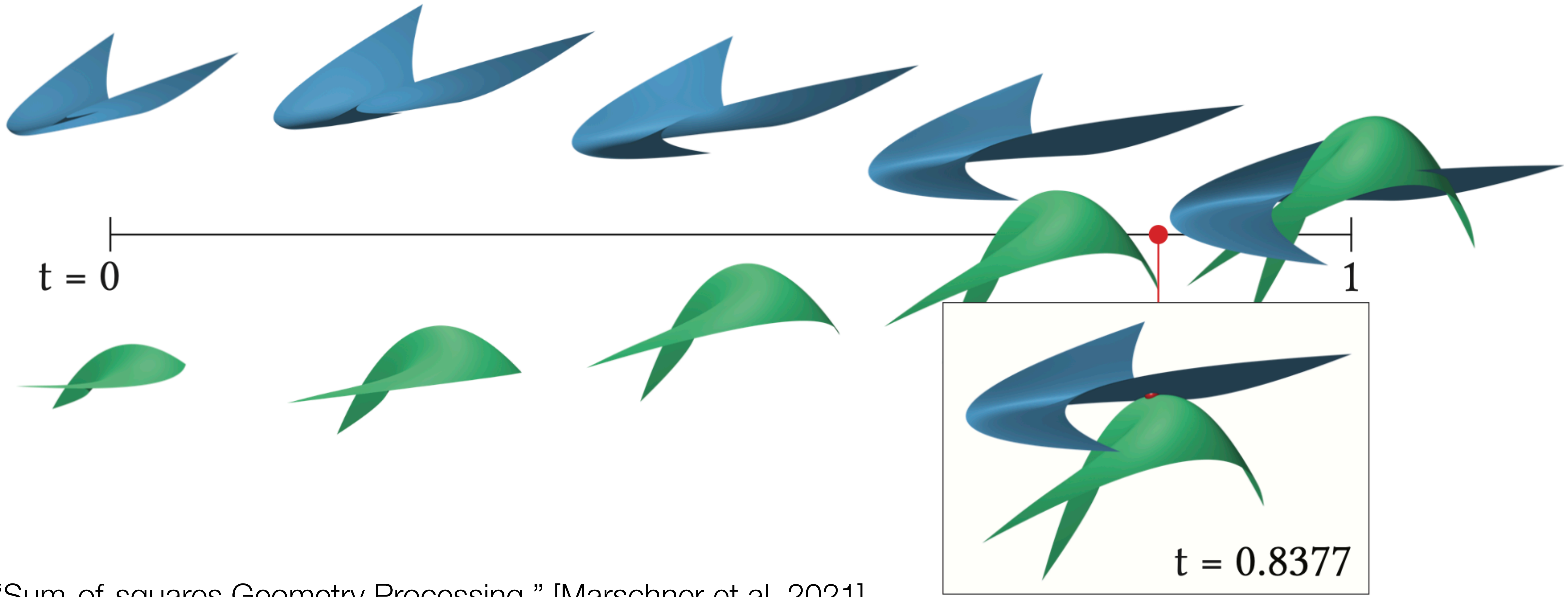


image borrowed from Doug James

Version 2: "If and when do these two moving objects collide?"

- continuous collision detection (CCD)
- can guarantee you don't miss collisions



"Sum-of-squares Geometry Processing." [Marschner et al. 2021]

Collision detection overview

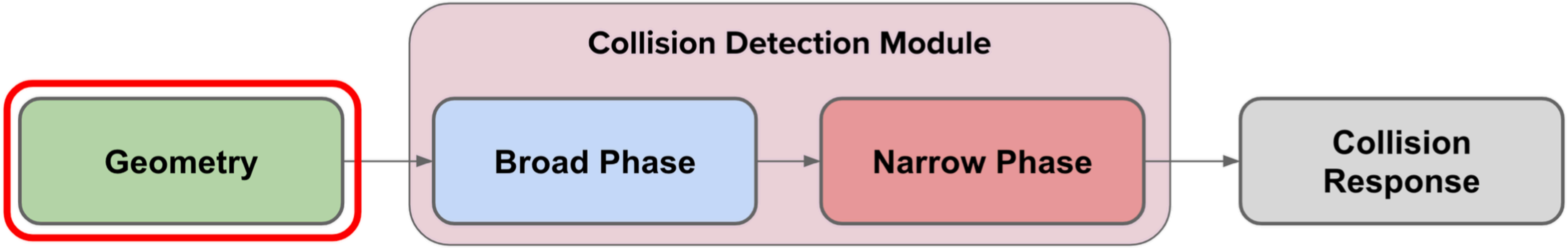
note: there's some disagreement between sources about where the boundary between "broad" and "narrow" goes…

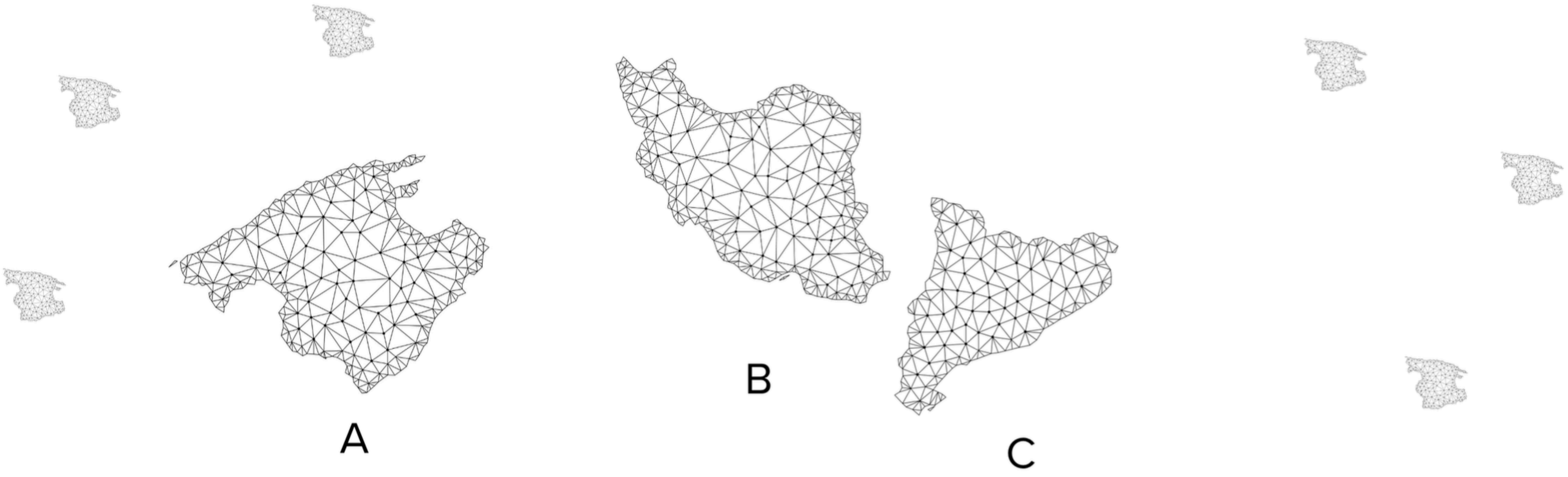
Narrow phase collision detection

- detects collisions between individual primitives
- produces definitive answers depending on the goals
    - yes/no for collision or proximity
    - time of collision
    - *k* nearest neighbors
- specific methods depend on primitive type (particles, lines, triangles, etc.)

Broad phase collision detection

- conservatively eliminates potential collisions
- reduces the set of narrow-phase tests required
- uses various spatial data structures for efficiency
- specific methods depend on data structure (trees, grids, lists, etc.)





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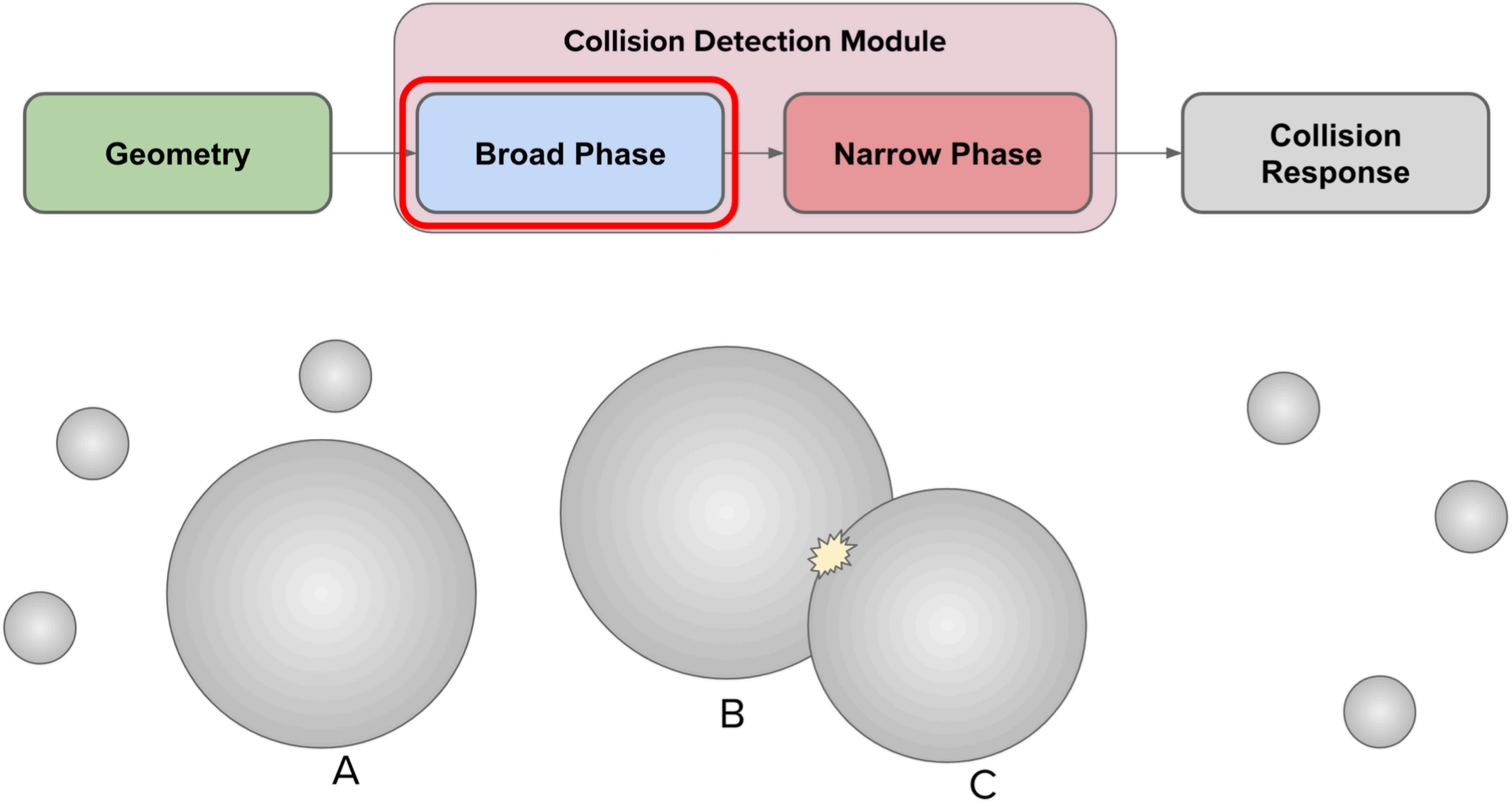

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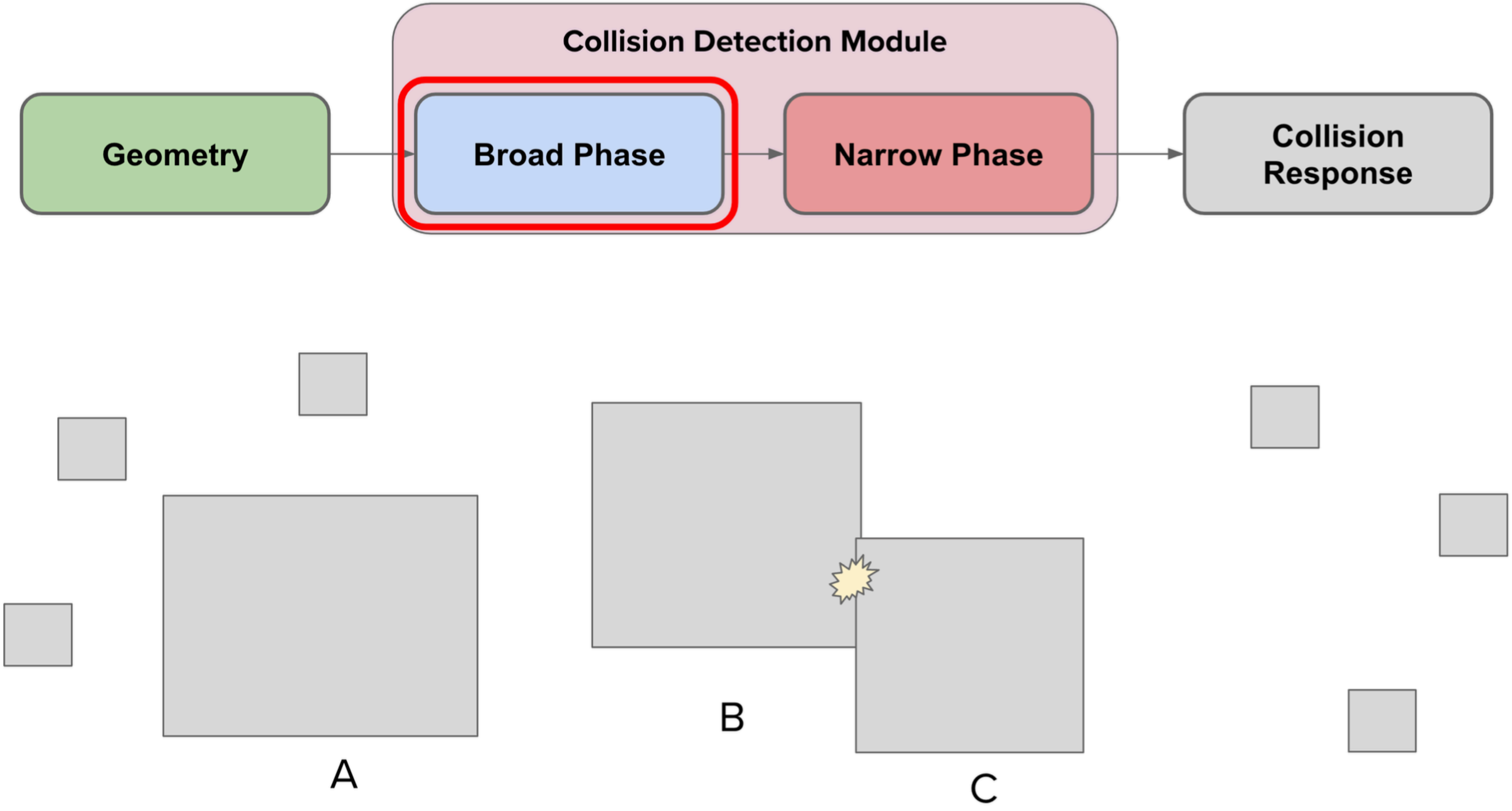

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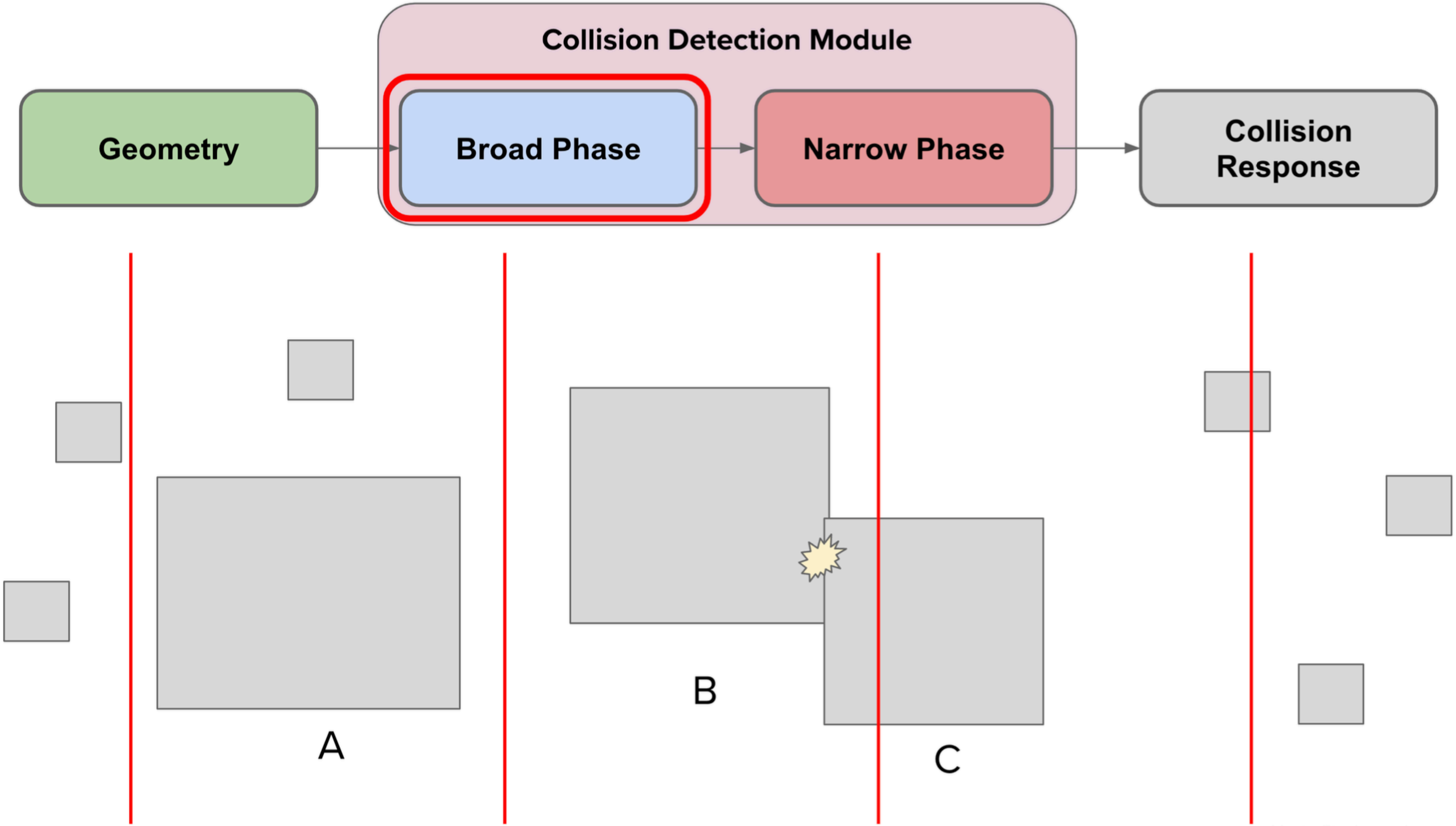

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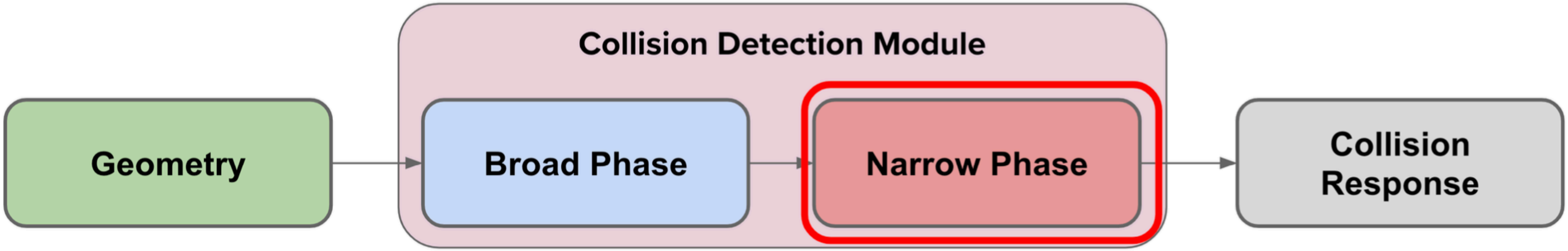

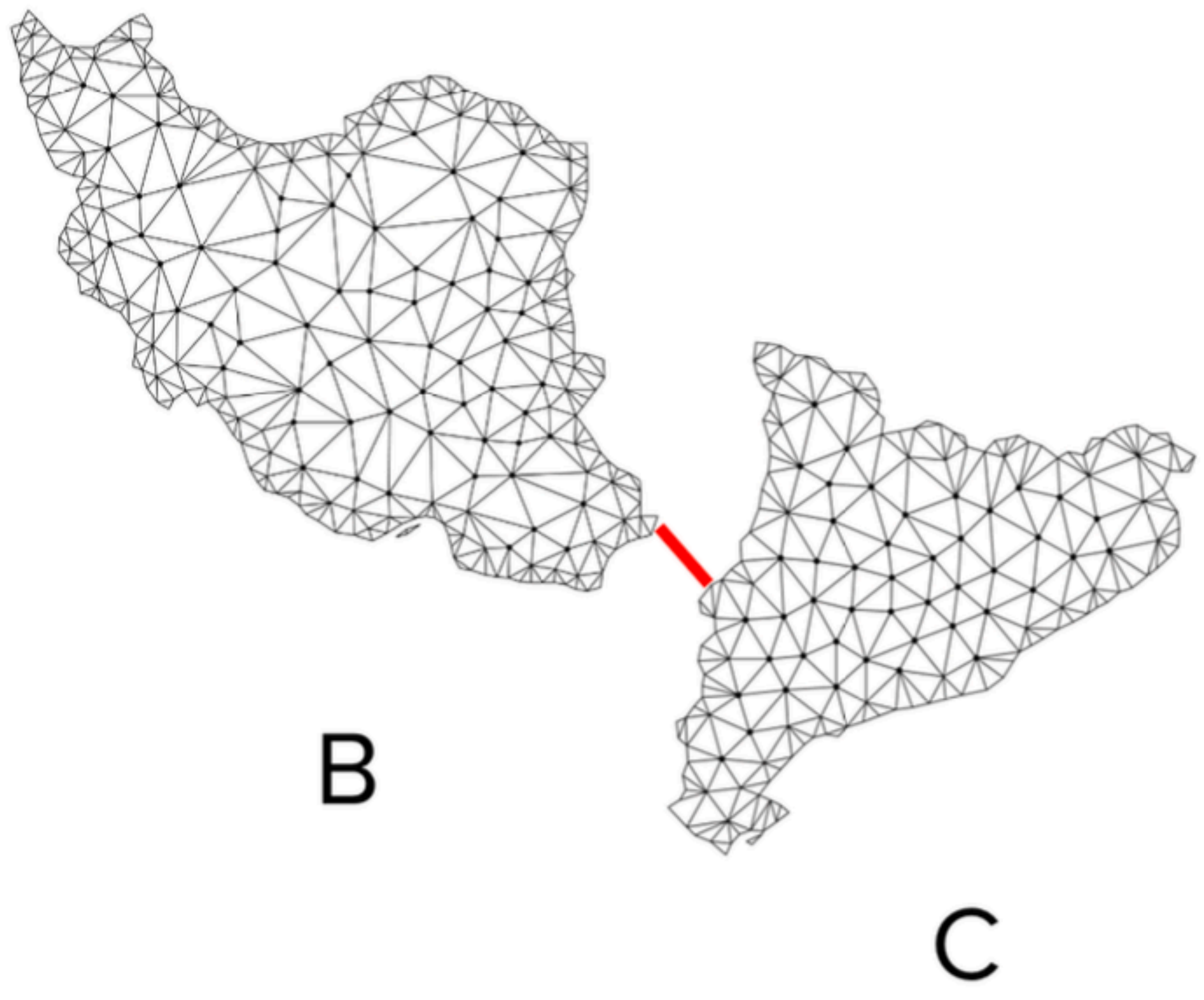

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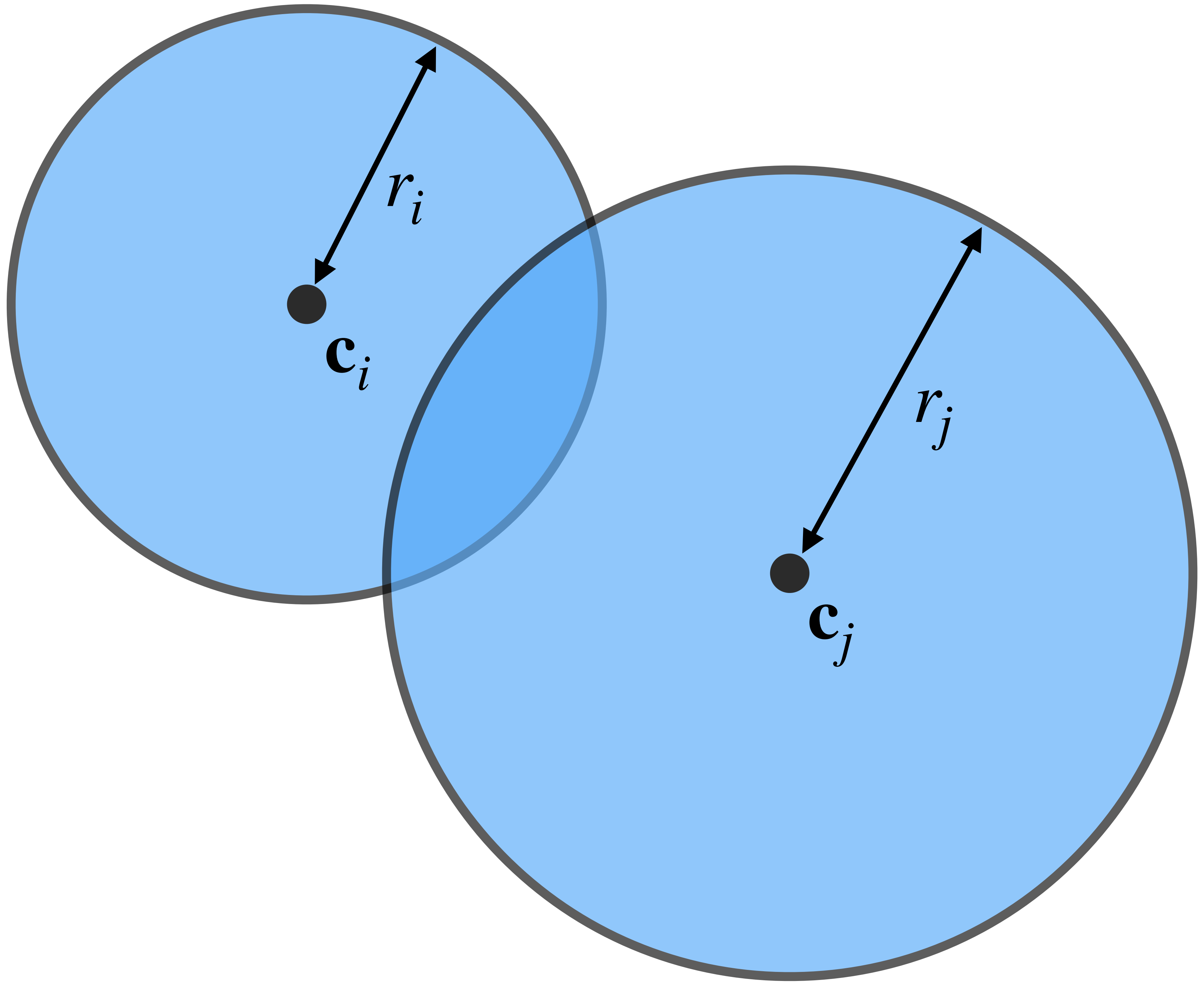
Simple narrow-phase example

Colliding spheres

- example for now, will return to more interesting cases
- spheres or circles intersect if $\|\mathbf{c}_i - \mathbf{c}_j\|^2 < (r_i + r_j)^2$



Broad phase algorithm #0

Brute force loop over all pairs

- problem: $O(N^2)$

```
for i in range(N):
    for j in range(N):
        CheckCollision(i, j)
```

Avoiding N^2

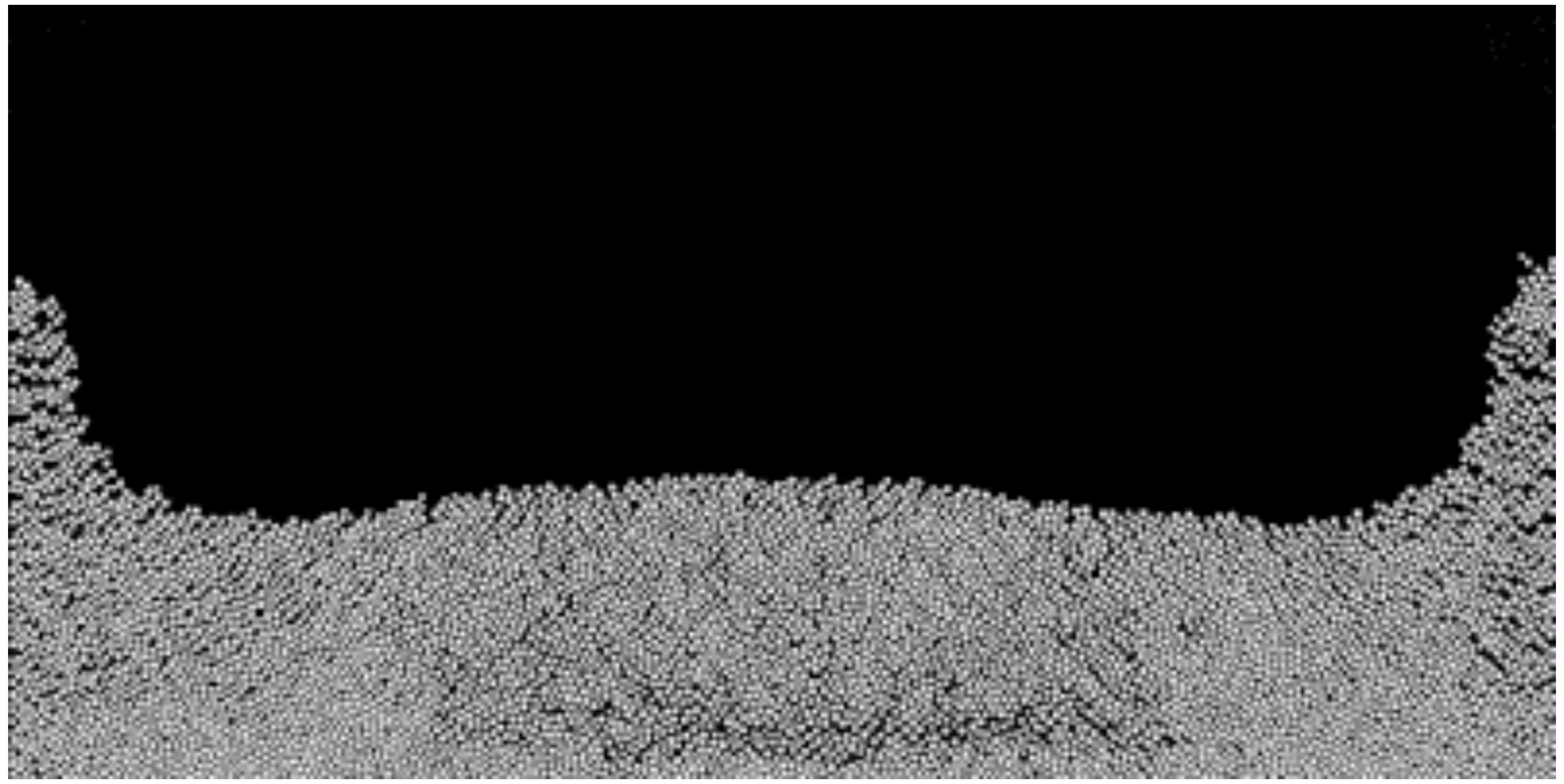
Sometimes there really are N^2 interactions

- have to deal with it
- reduce to $O(N)$ or $O(N \log N)$ by hierarchically approximating distant interactions
  - Fast Multipole Method (FMM)
  - Barnes-Hut approximation

In simulations usually only neighboring objects interact

- actual number of contacts is probably $O(N)$ for N objects
- goal is to efficiently search for “active contacts”

InsideHPC

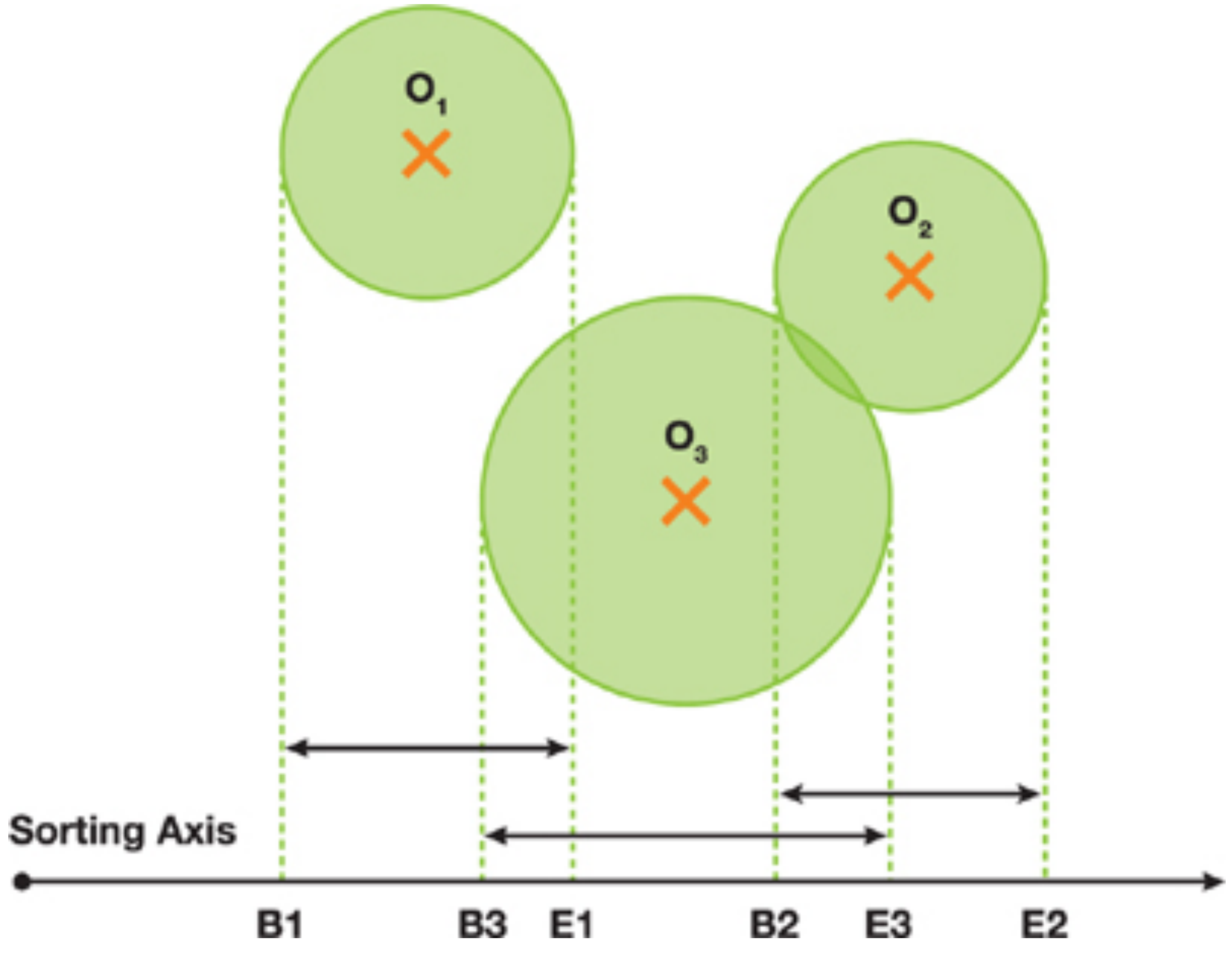


Taichi DEM demo

Collision detection by sort / sweep

Older idea: sort and sweep

- choose an axis (call it *x*) and project objects onto it
- put the min (begin) and max (end) *x* coordinates for each object into a big list
- sort the list
- traverse the list
  - begin object *i* -> add object *i* to active set
    check object *i* against active set
  - end object *i* -> remove object *i* from active set



Scott Le Grand, GPU Gems 3 Chapter 32

Problems

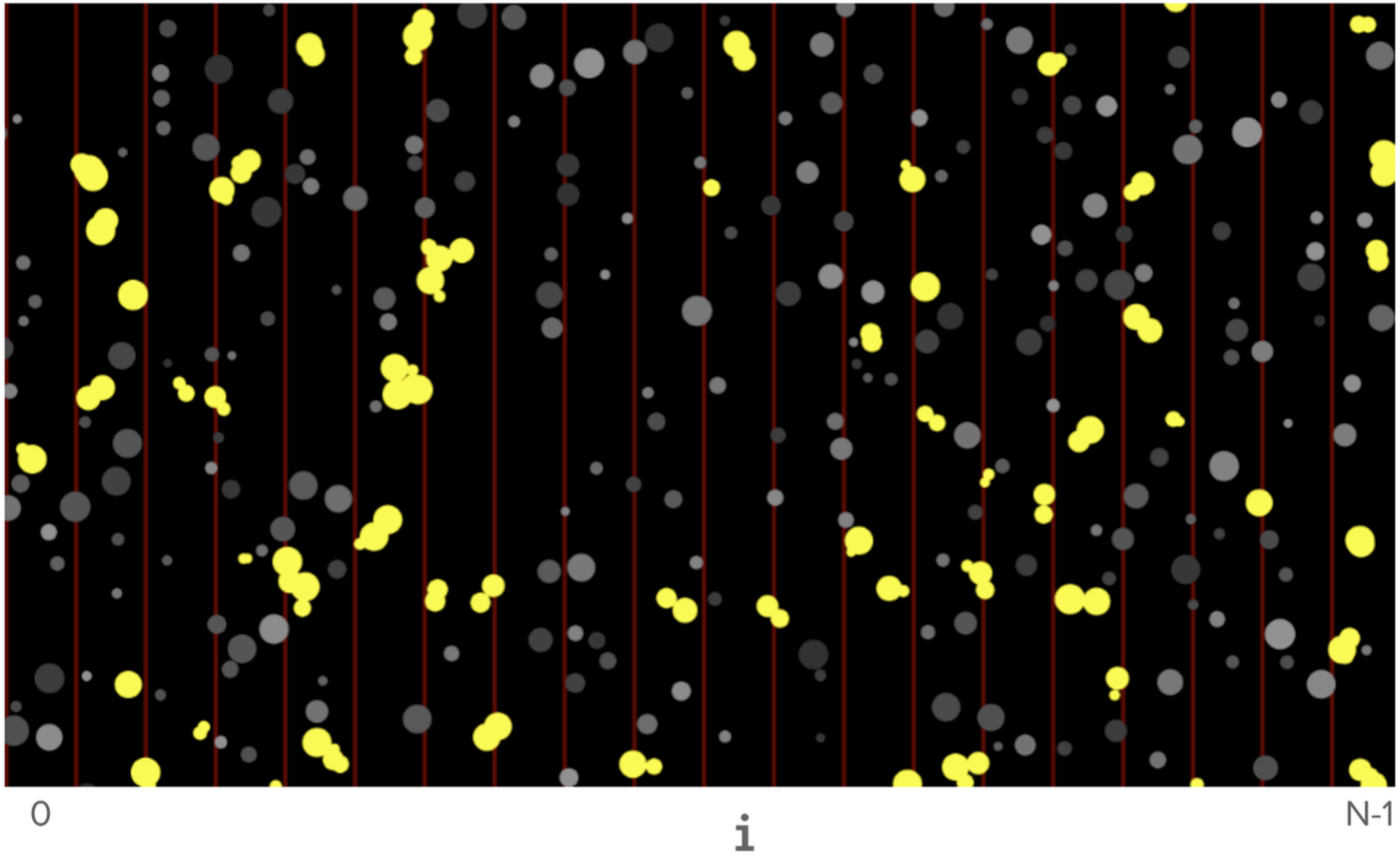
- sorting is not so parallel friendly
- what is the worst case for this? what is the time complexity for uniformly distributed objects?

Regular grid broad phase: 1D subdivision

Construction:

- Divide space into N bins of equal width, h
- Add each object to each bin that its bounding volume overlaps:
  - Use 1D overlap test

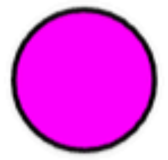
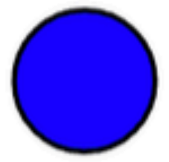
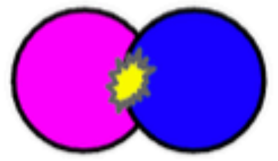
Cell Index, i: Given coordinate x, find containing cell `index(x)` using `Math.floor(x/h)` clamped to [0,N-1].

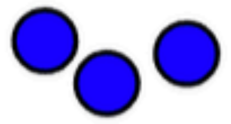


slide borrowed from Doug James

Regular grid broad phase: 1D subdivision

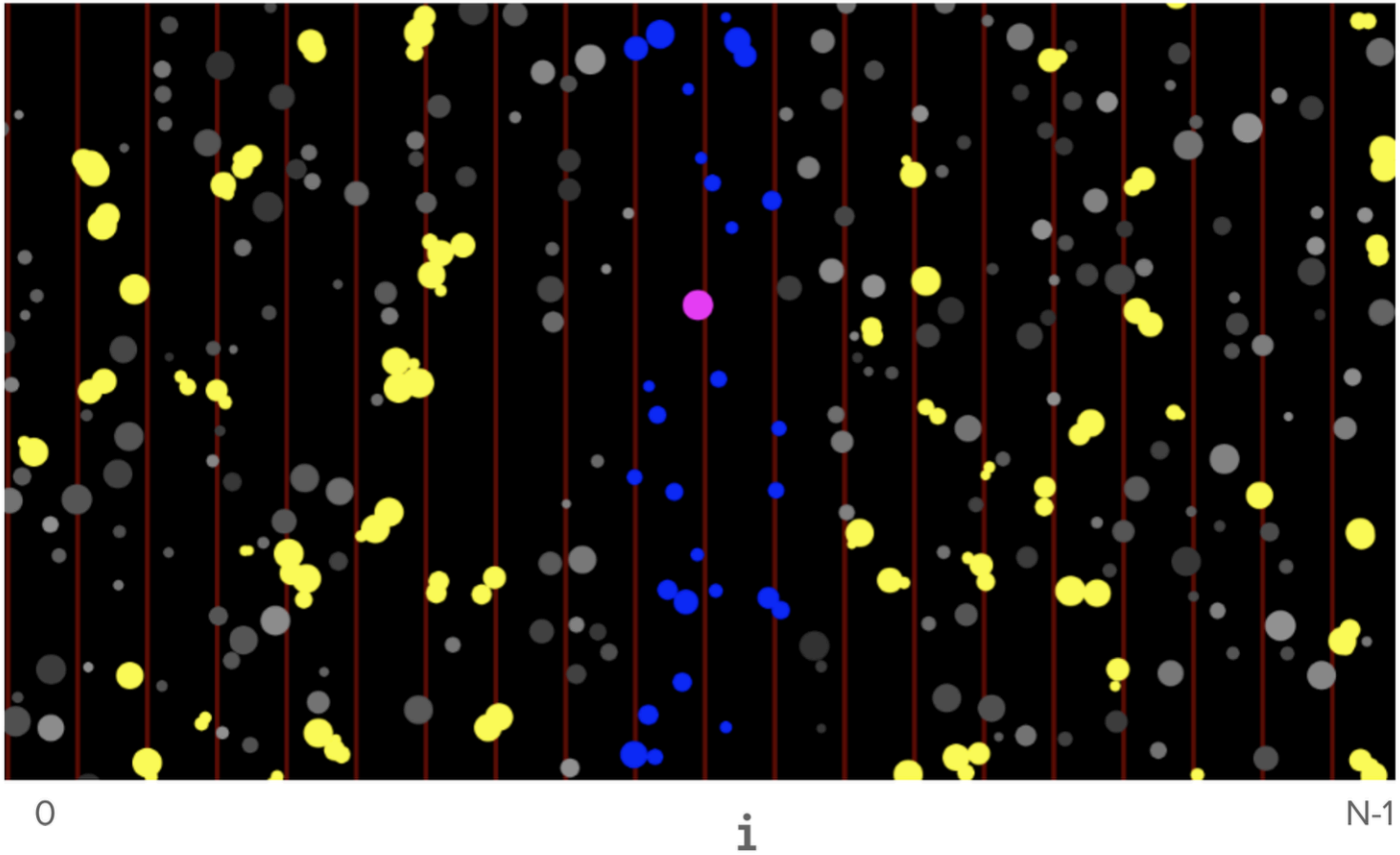
Overlap Testing:

- Given test bound 
- Find overlapping cells, and for each bound 
    - Do overlap test 
- Return overlapping results as a set.



Q: Can duplicate overlaps occur?

Weakness of 1D subdivision?



slide borrowed from Doug James

Regular grid broad phase: 2D subdivision

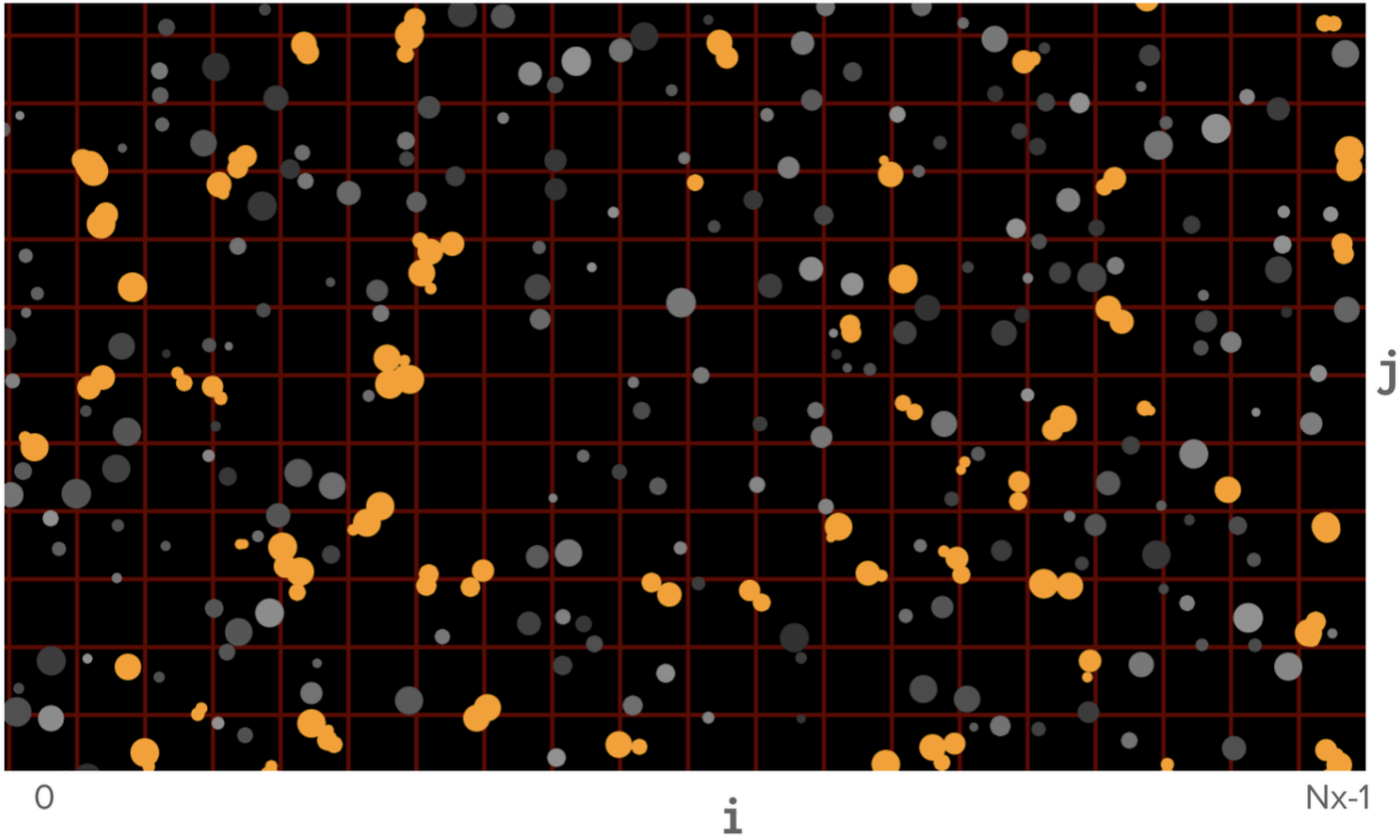
Construction:

- Divide space into Nx-by-Ny bins of constant width, h (or hx & hy)
- Add each object to each bin that its bounding volume overlaps:
  - Use 1D overlap tests

Cell Index (i,j): Given coords x & y,

`i = floor(x/h`$_x$`)` clamped to [0,Nx-1],

`J = floor(y/h`$_y$`)` clamped to [0,Ny-1].



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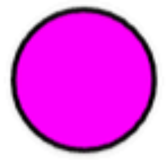
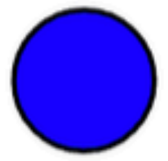
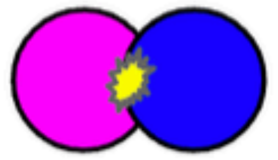
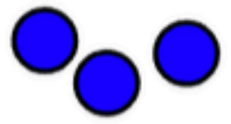
Regular grid broad phase: 2D subdivision

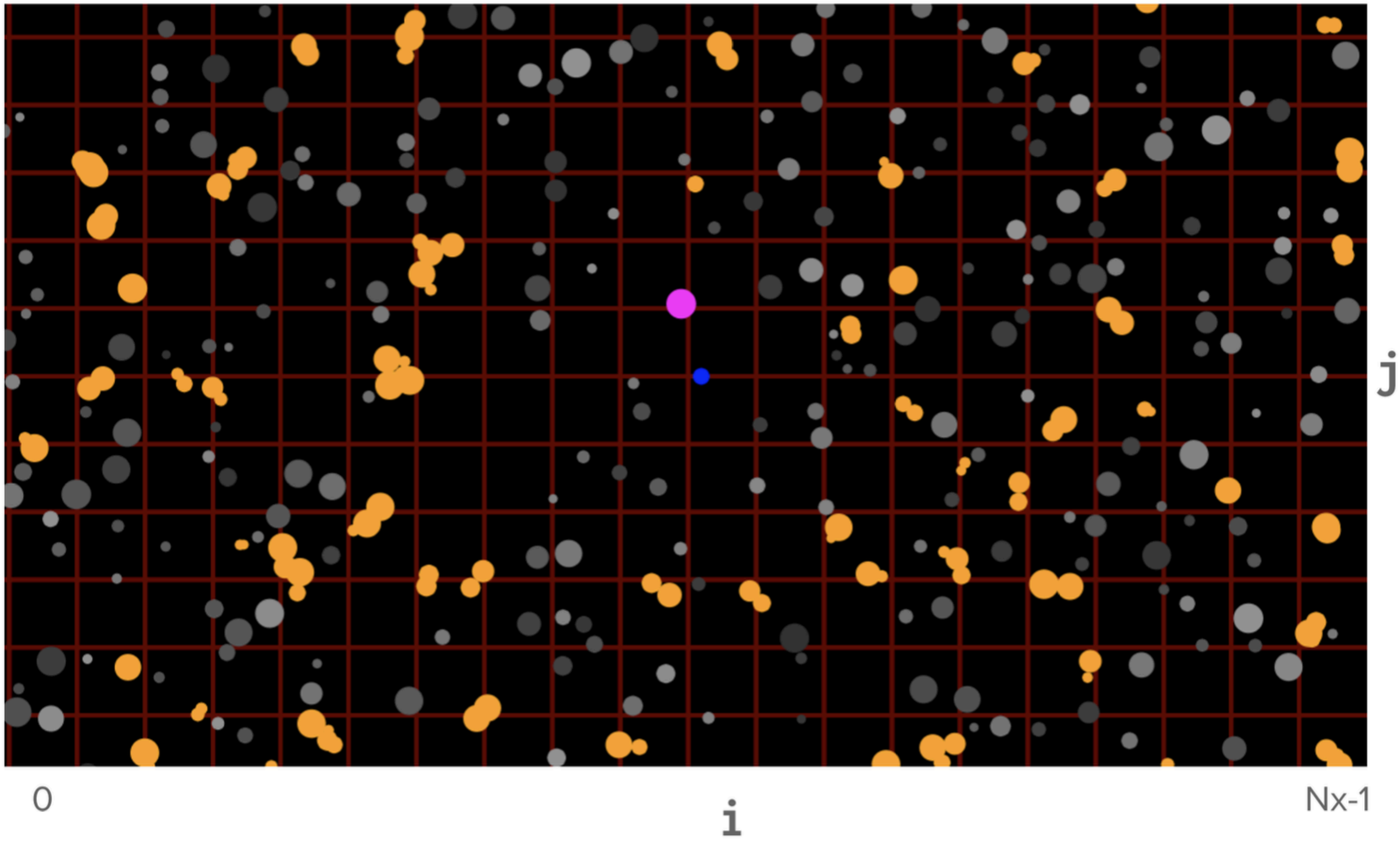
Overlap Testing:

- Given test bound 
- Find overlapping cells, and for each bound 
  - Do overlap test 
- Return overlapping results as a set.



Q: Can duplicate overlaps occur?

Weakness of 2D subdivision?



slide borrowed from Doug James

2D spatial subdivision

demo by Doug James

Advantages [demo]

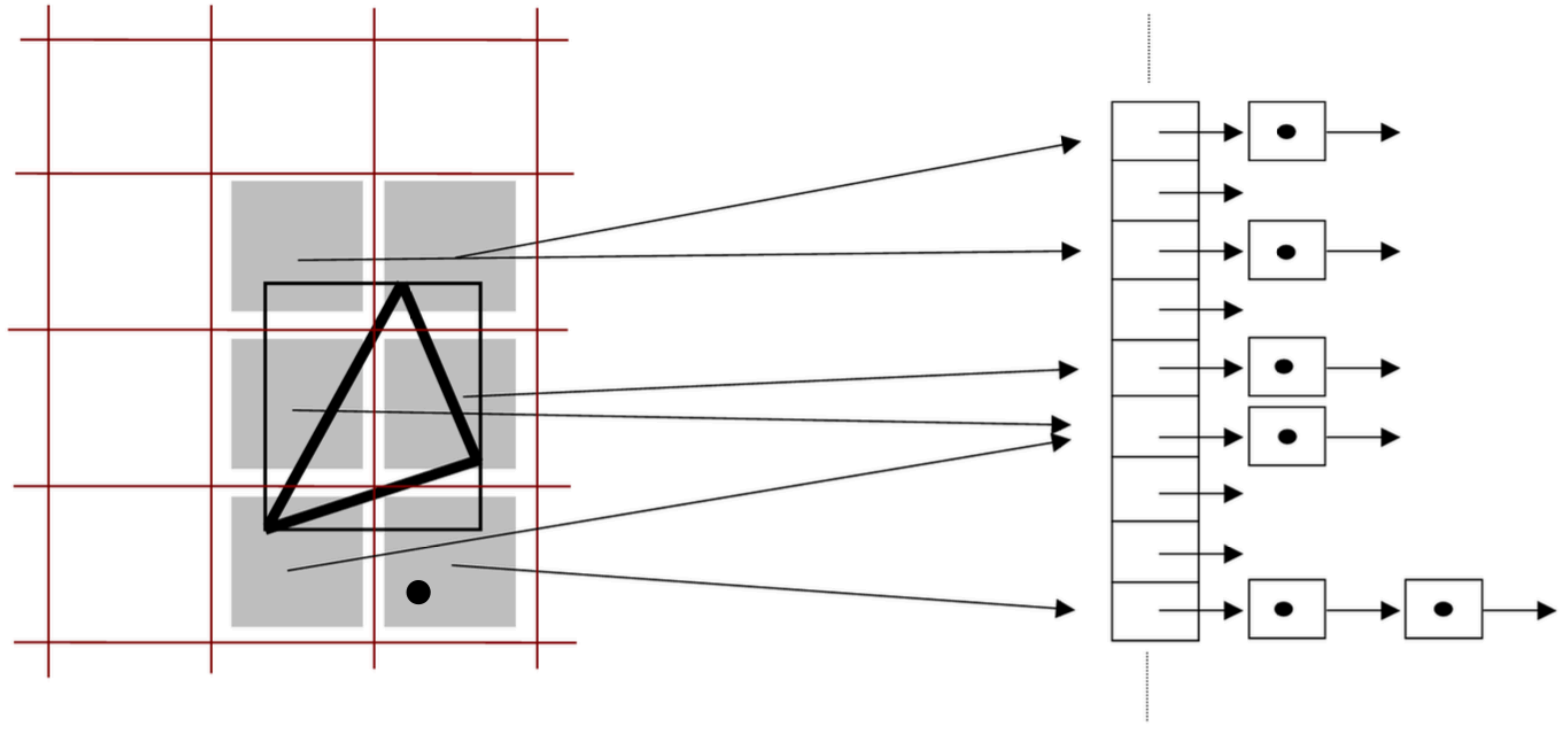
- often quite efficient; fairly simple to implement; reasonably parallel-friendly

Disadvantages

- large tables of possibly mostly empty particle lists; need to set grid dimensions up front
- what are the cases where it gets slow?

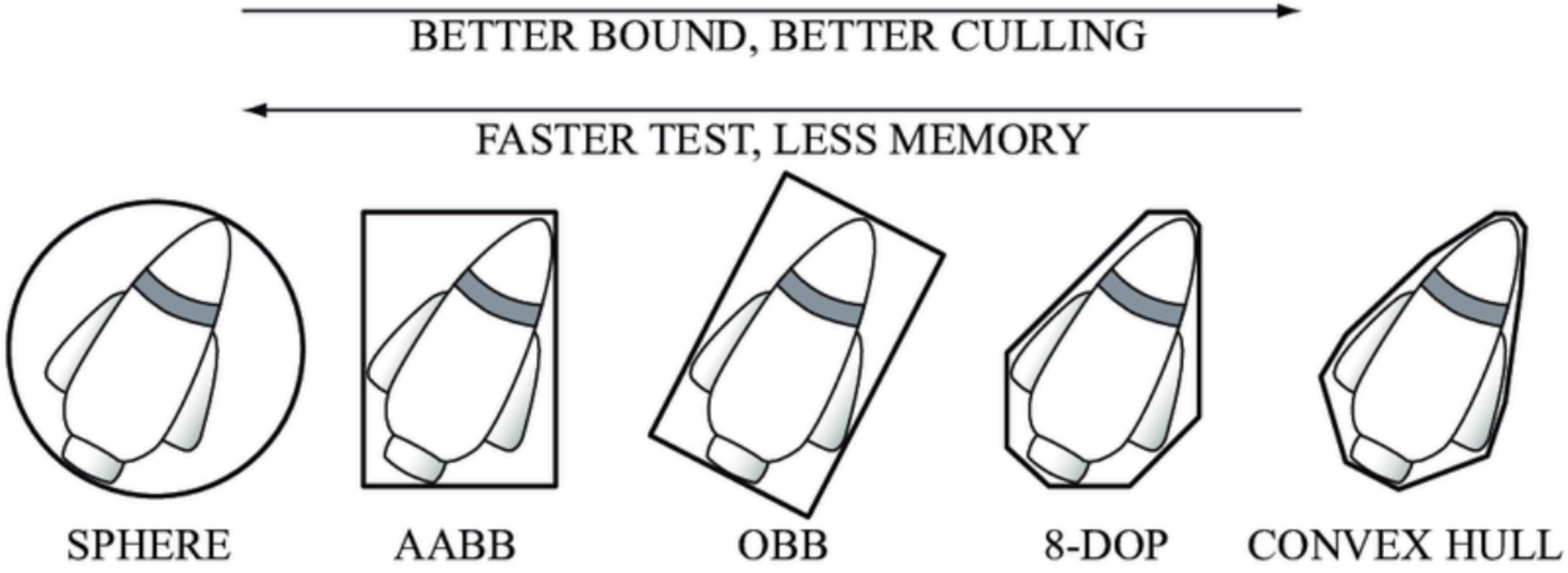
Variations

- spatial hashing: rather than grid[x,y],
  use table[hash(x,y)] for a suitable hash function
  - allows effectively unlimited grid; hash collisions
    just lead to some extra collision tests
- quadtrees, octrees: allow balancing cell occupancy
  when objects are nonuniformly distributed



Teschner et al 2003

Bounding volumes



[Ericsson 2004]

Simple idea to speed up collision checks

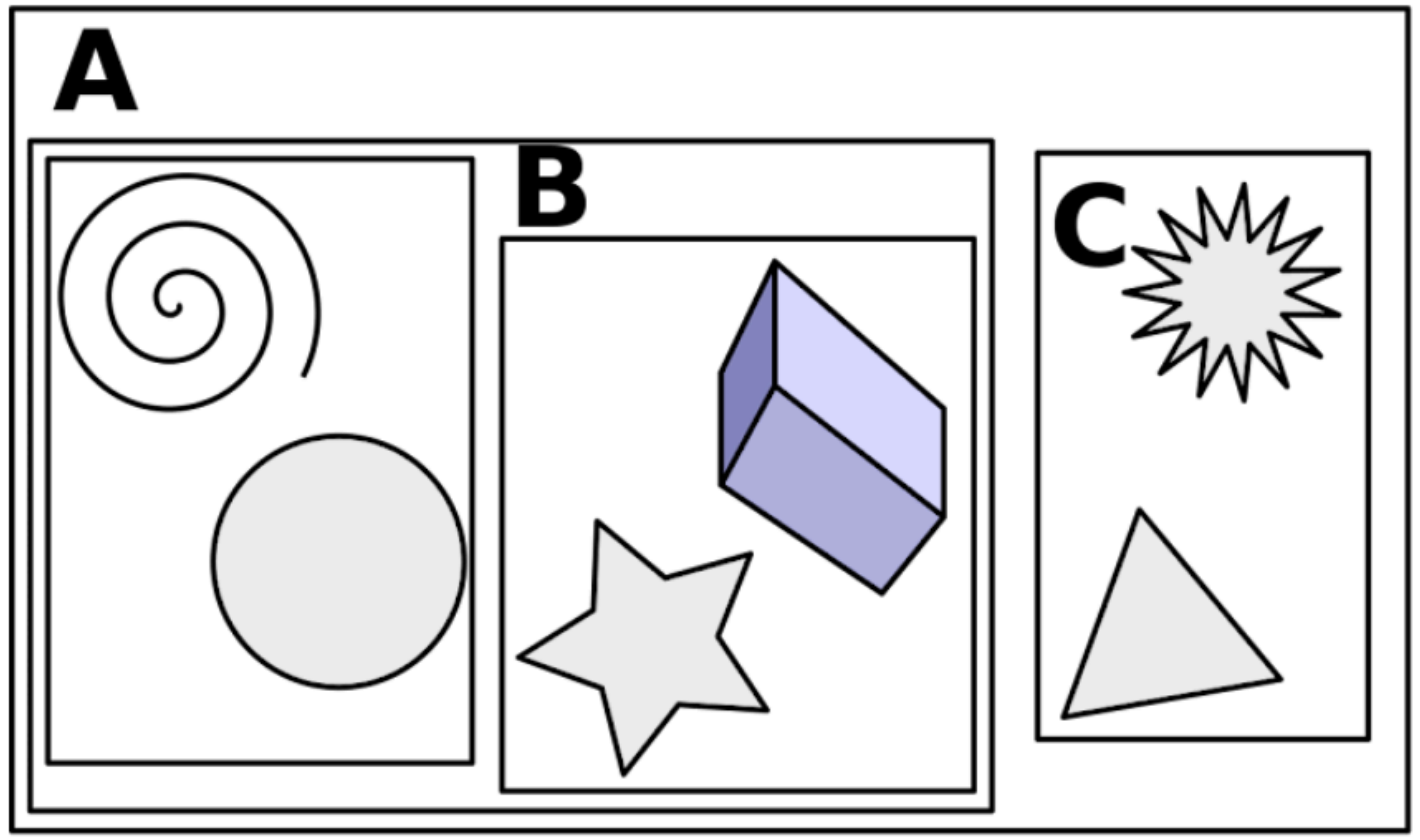
- first find a volume that contains (bounds) each object
- then when you want to test two objects for collision, first check whether their bounding volumes intersect
- no BV intersection → no collision, *guaranteed*!
- BV intersection → no guarantee, need to check for collisions
- for efficiency of intersection testing, BVs are always **convex**

Bounding volume hierarchies

Similar to those used for ray intersection

- can use any sort of bounding volume (BV)
- for any collision test, if the BV does not collide then the entire subtree can be skipped
- algorithms differ depending on query type
- to test against a simple obstacle for which a fast test is available, a simple traversal does the trick:

```
overlap(node, obstacle):
   if overlap_bv(node.bounds, obstacle):
      if node.is_leaf():
         return overlap_geom(node.geom, obstacle)
      else
         return overlap(node.left, obstacle) or
            overlap(node.right, obstacle)
   return false
```



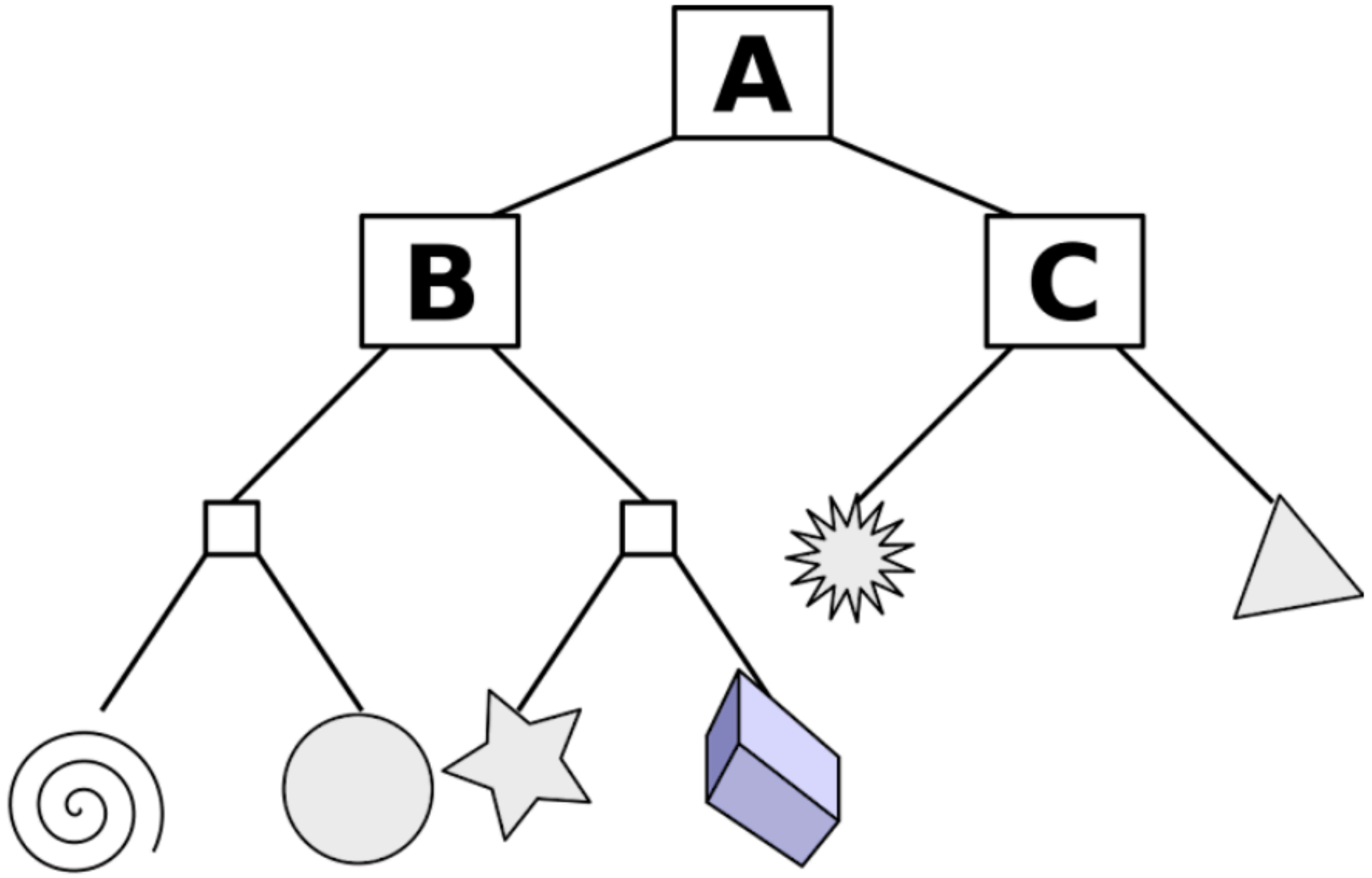
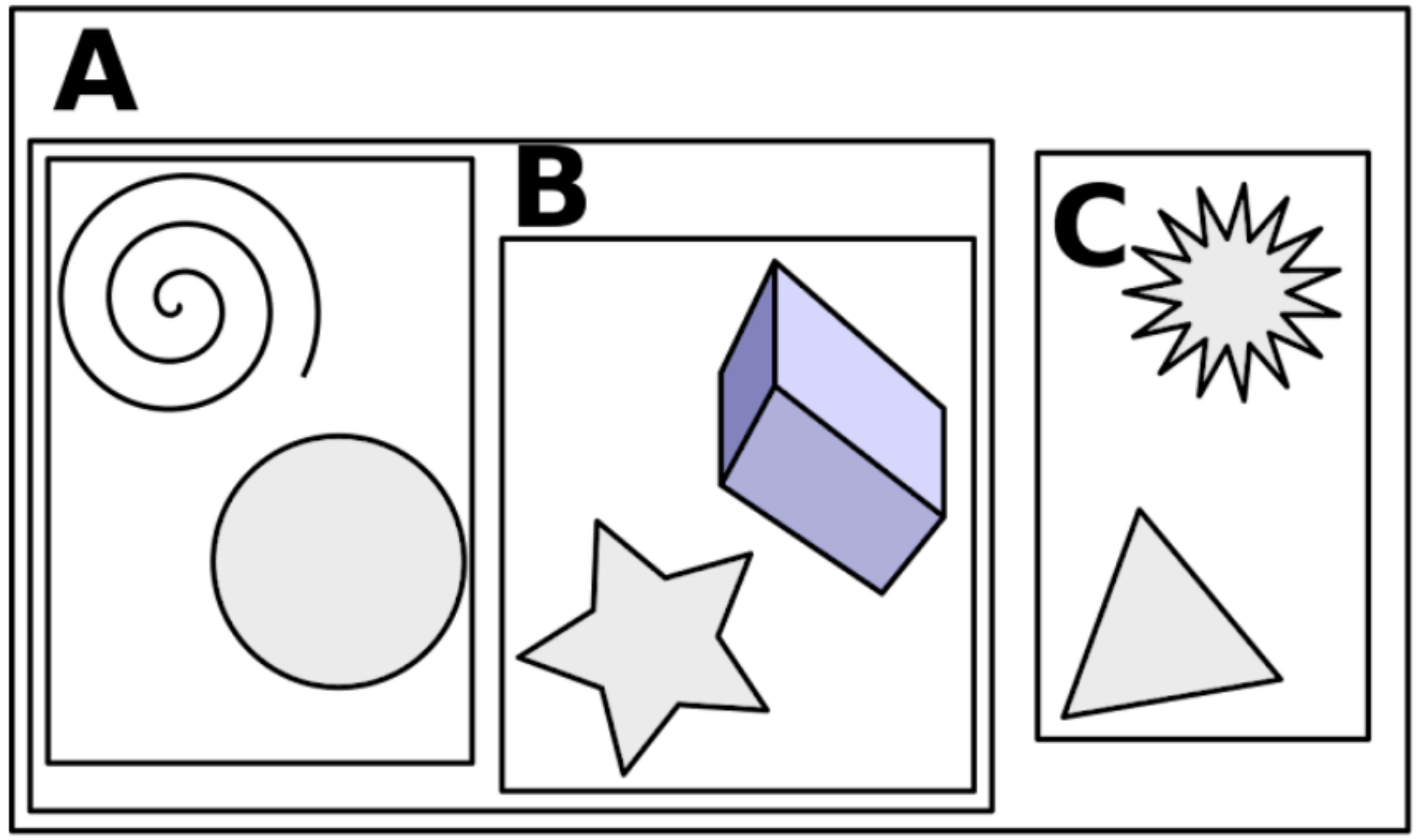


figure borrowed from Doug James

Bounding volume hierarchies

- to test against another complex object with its own BVH hierarchy, traverse trees in tandem:

```
overlap(node1, node2):
  if overlap_bv(node1.bounds, node2.bounds):
    if node1.is_leaf() and node2.is_leaf():
      return overlap_geom(node1.geom, node2.geom)
    if node1.is_leaf():
      return overlap(node1, node2.left) or
        overlap(node1, node2.right)
    if node2.is_leaf():
      return overlap(node1.left, node2) or
        overlap(node1.right, node2)
    if node2.long_axis() > node1.long_axis():
      return overlap(node1, node2.left) or
        overlap(node1, node2.right)
    else
      return overlap(node1.left, node2) or
        overlap(node1.right, node2)
  return false
```



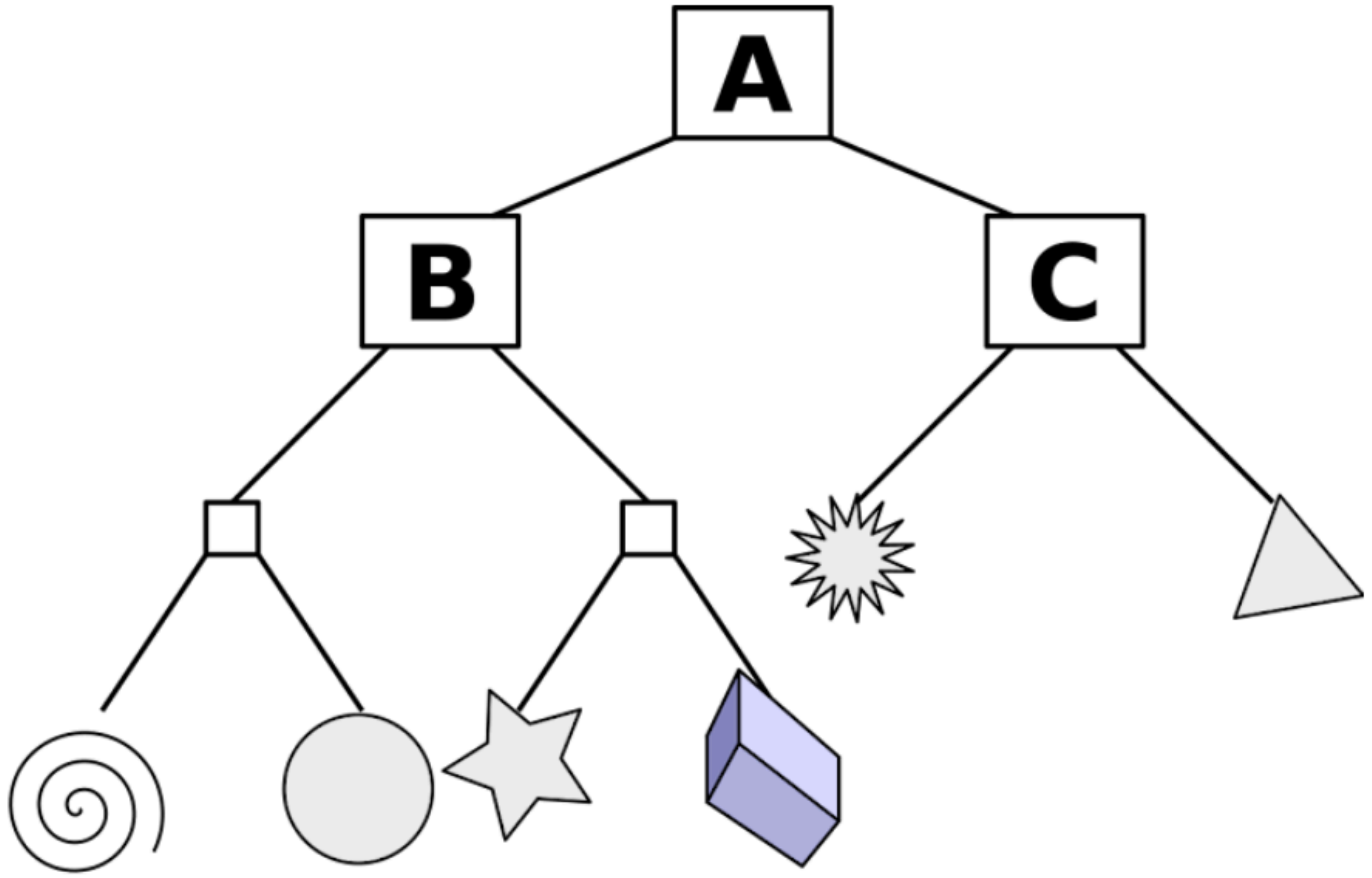
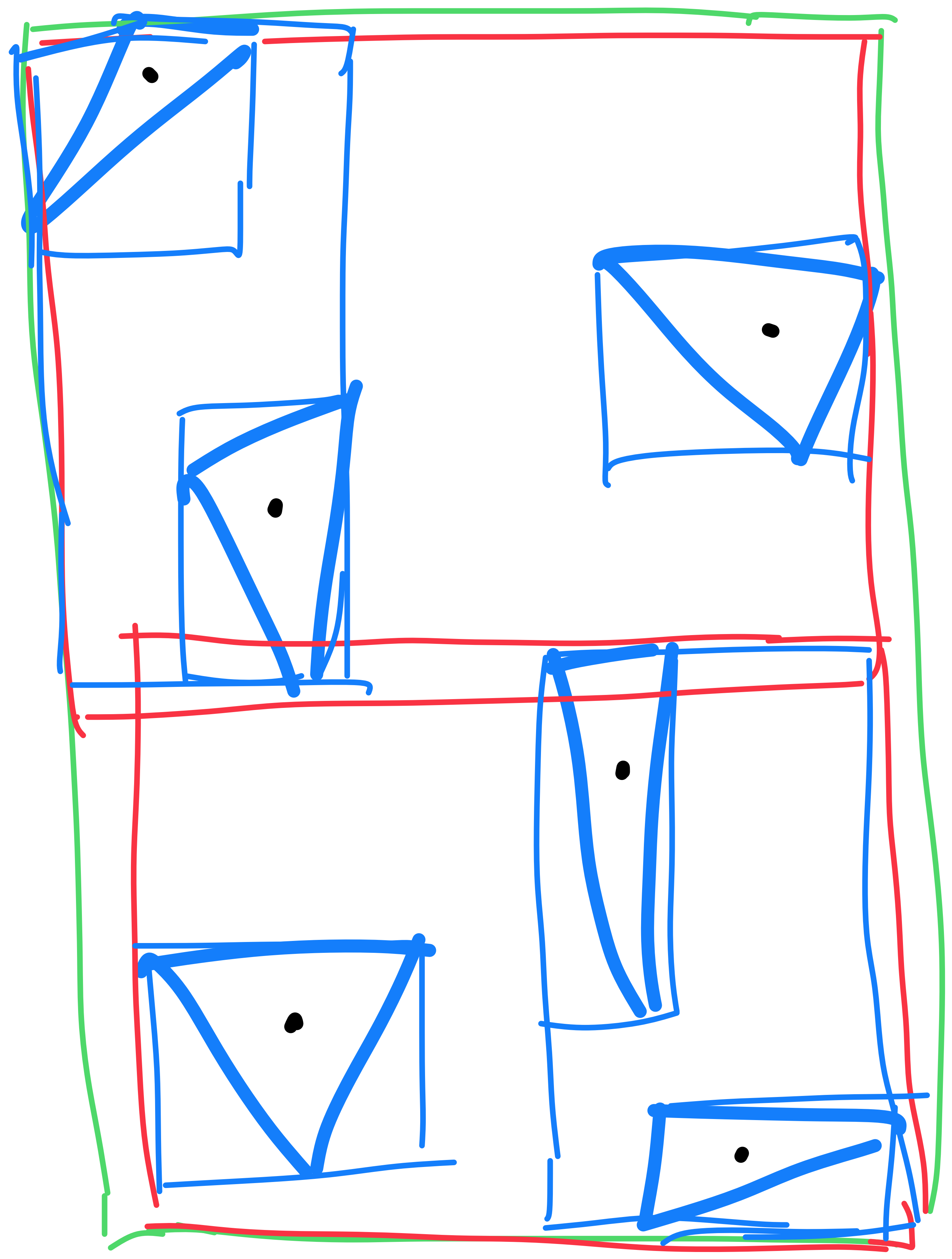


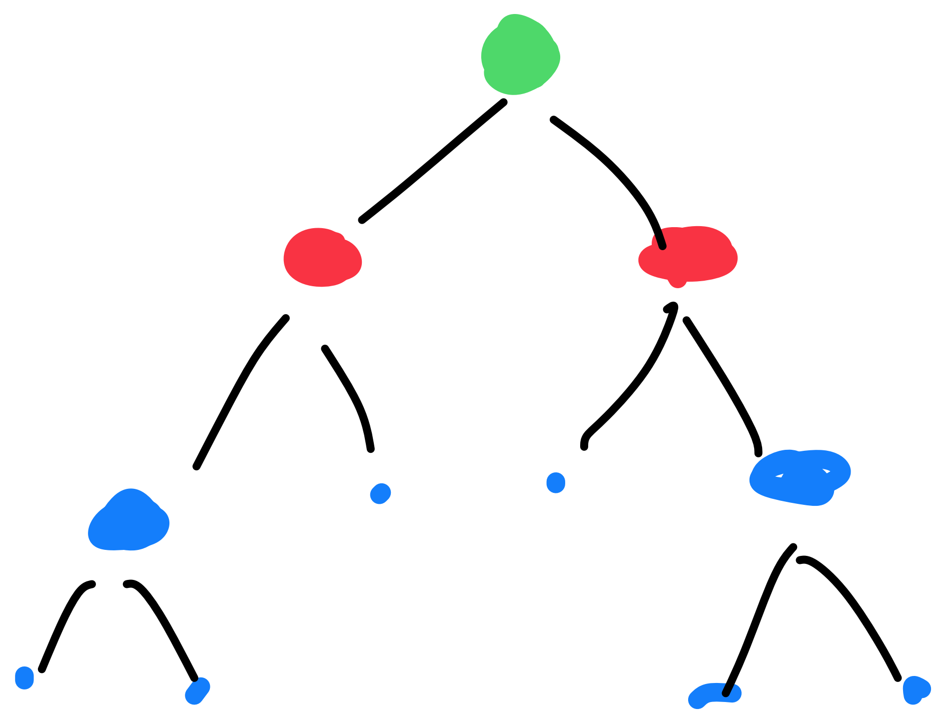
figure borrowed from Doug James

Building BVHs

Simplest way: top down splitting

- fit BV to all the geometry you have
- split geometry into two equal sized subsets
  - simple strategy: median split
  - choose axis along which to split
    (typically the longest BV axis)
  - split at median of projections of object
    centroids onto that axis
- recursively process the two halves

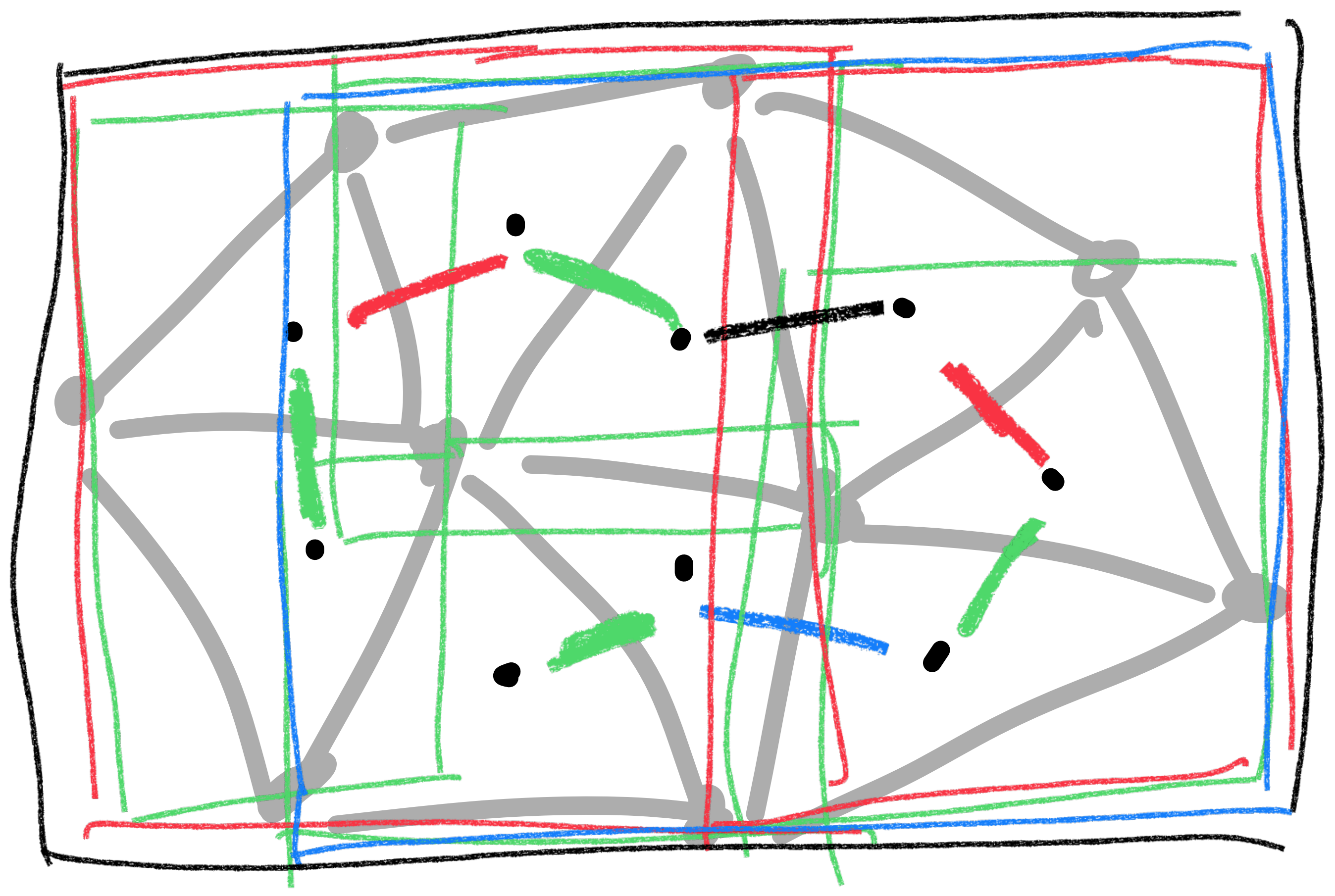


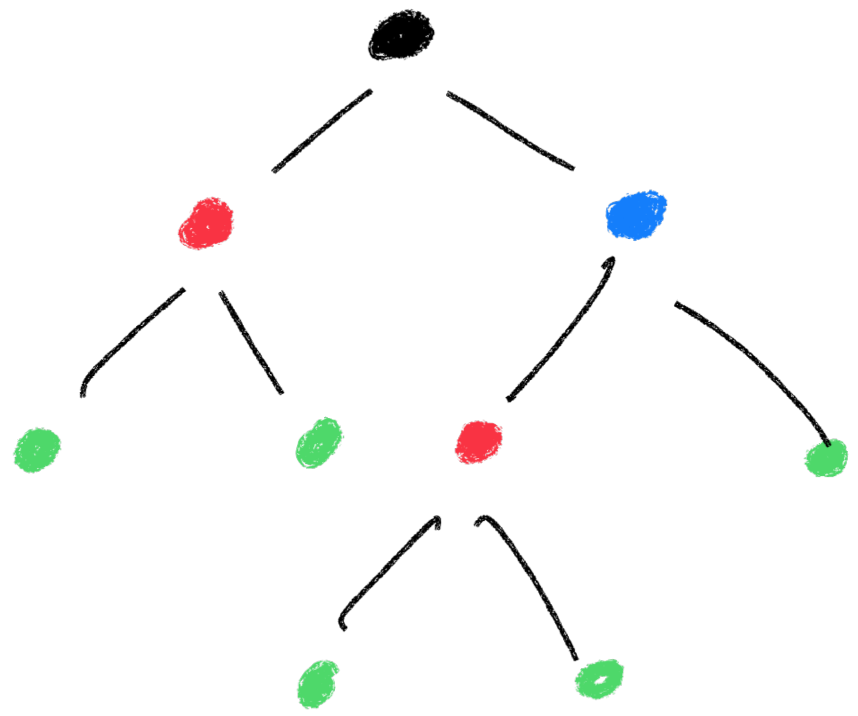


Building BVHs

Splitting according to mesh connectivity

- might want nodes to contain contiguous parts of objects
- leads to a bottom-up approach
  - build an adjacency graph of all primitives
  - repeatedly choose an edge with lowest "cost" and merge the two nodes
  - cost might be the volume of the resulting node or the height of the resulting subtree
- popular for deformables, produces trees likely to re-fit well (next slide)

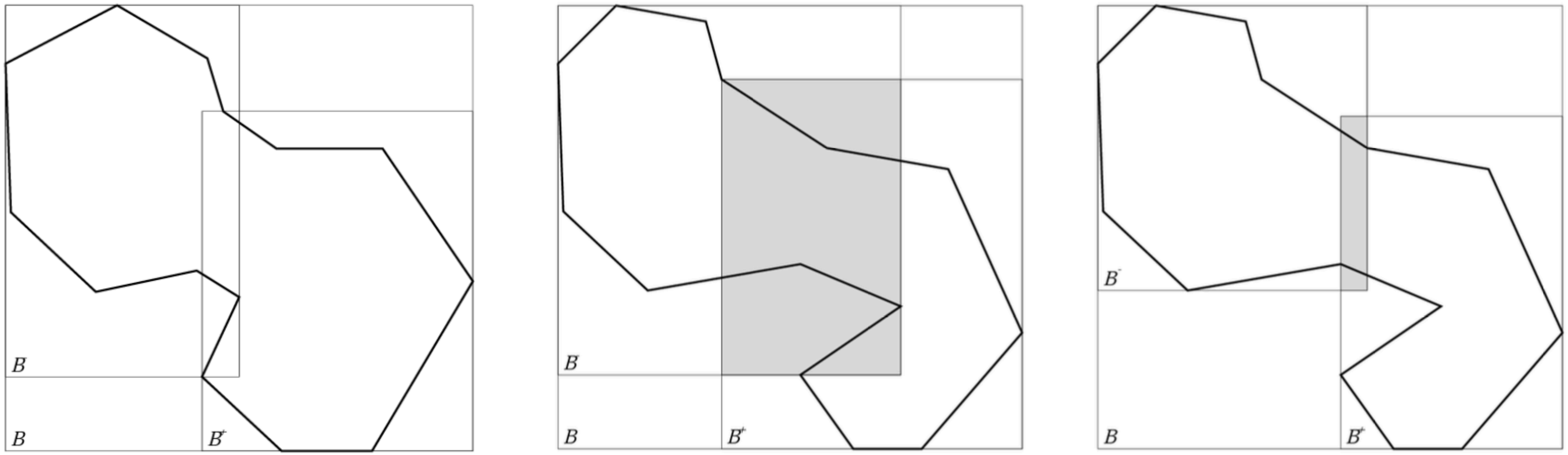




Updating BVH for deforming geometry

Geometry is different each frame—what to do?

- constructing a new tree from scratch every frame is expensive
- alternative: keep tree structure and re-fit bounds
  - simple bottom-up algorithm with reasonable memory access pattern
  - efficient for BVs that can efficiently bound their children
  - downside: can lead to increased overlap; mesh connectivity ameliorates this



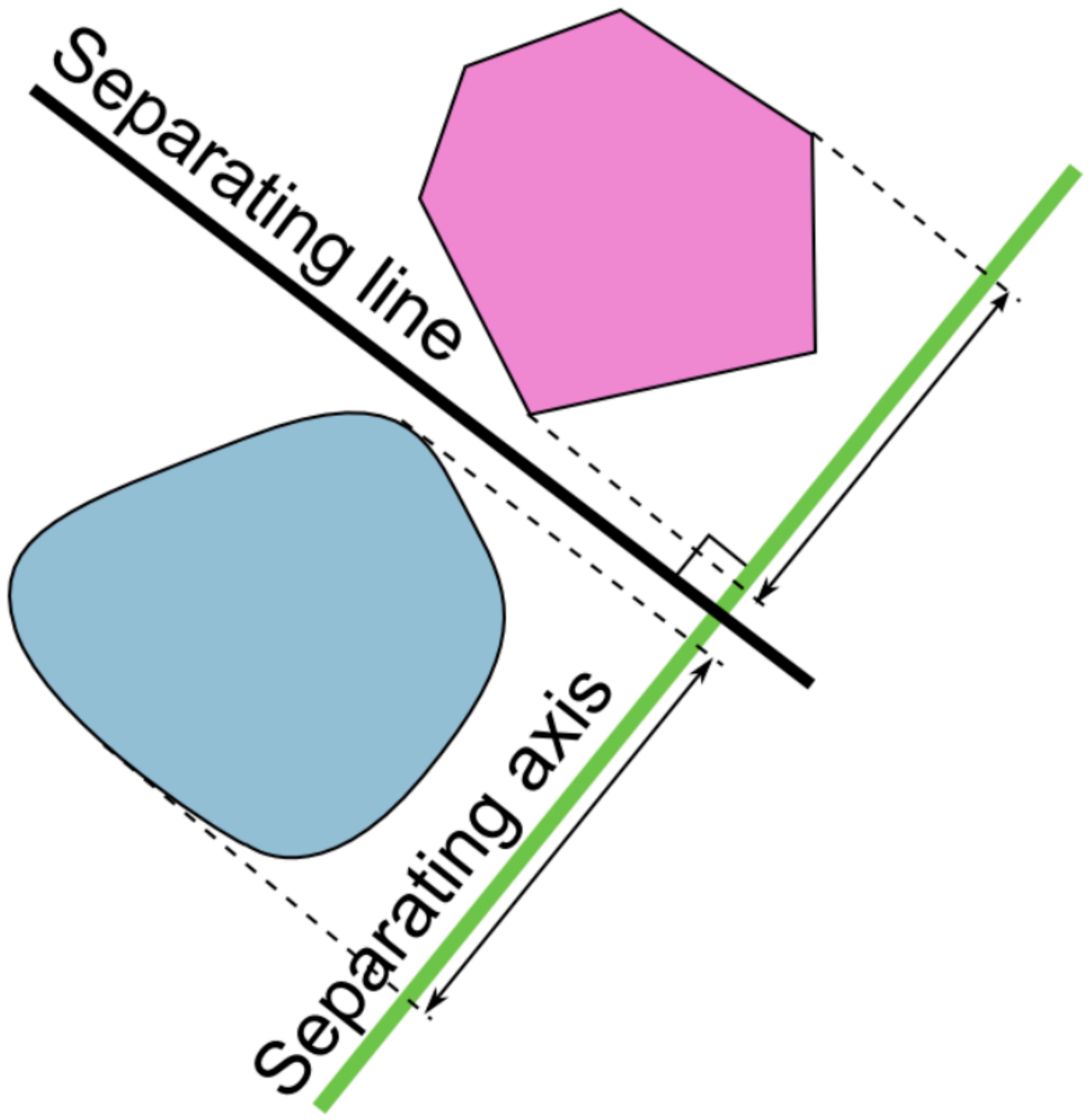
Undeformed (a) Refi tted (b) Rebuilt

[Gottschalk et al. 1996]

Finding collisions between convex polyhedra

An efficient strategy for fast BV intersection

- if the projections of two objects onto some axis are disjoint, the objects do not intersect and the axis is a *separating axis*
- if the objects do not intersect, a separating axis must exist
- for convex polygons in 2D or polyhedra in 3D, if there is no intersection then checking a finite list of potential separating axes suffices

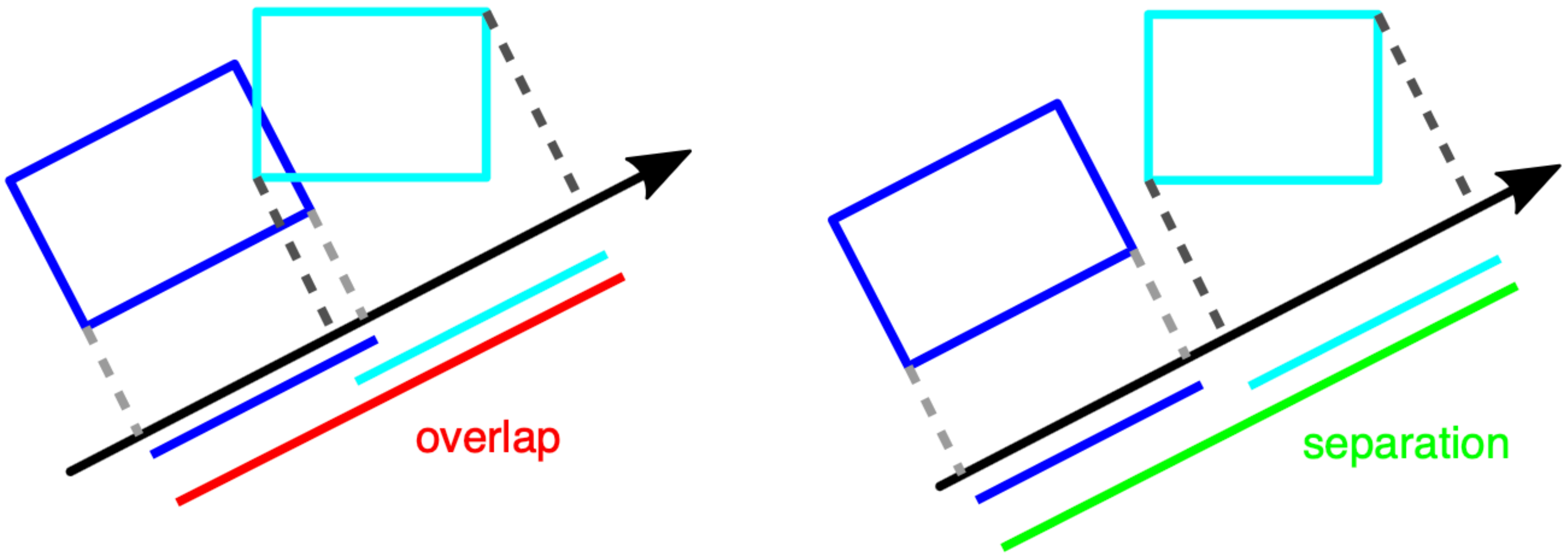


https://en.wikipedia.org/wiki/Hyperplane_separation_theorem

Examples

- 2 familiar tests for AABBs in 2D
- 4 tests for OBBs in 2D (4 distinct face normals)
- 15 tests for OBBs in 3D (6 face normals + 9 edge/edge normals)

E.g. separating axis approach for OBBs in 2D

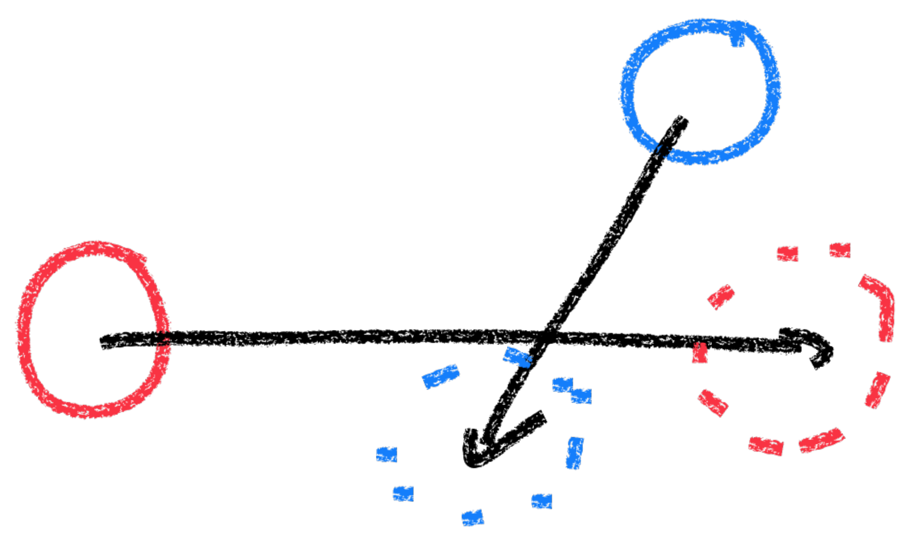


https://www.atoft.dev/posts/2020/04/12/implementing-3d-collision-resolution/

Continuous collision detection (CCD)

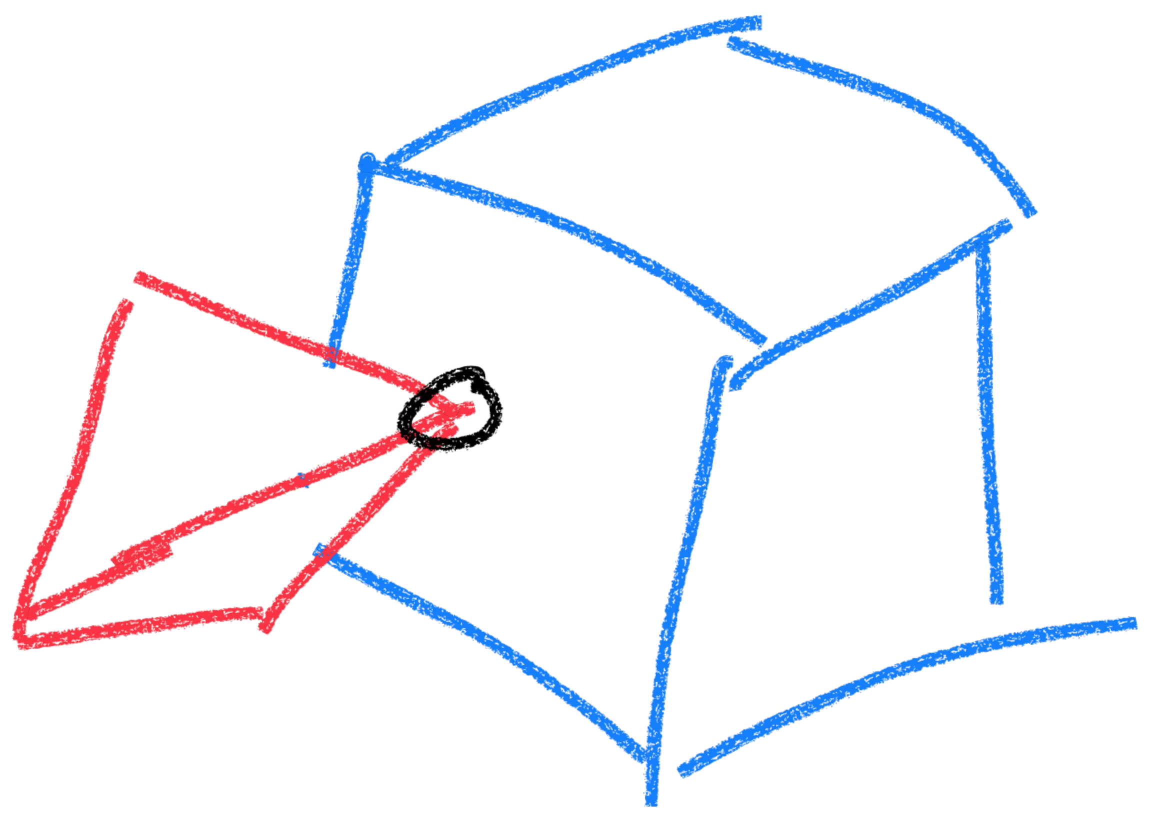
Given two moving primitives:

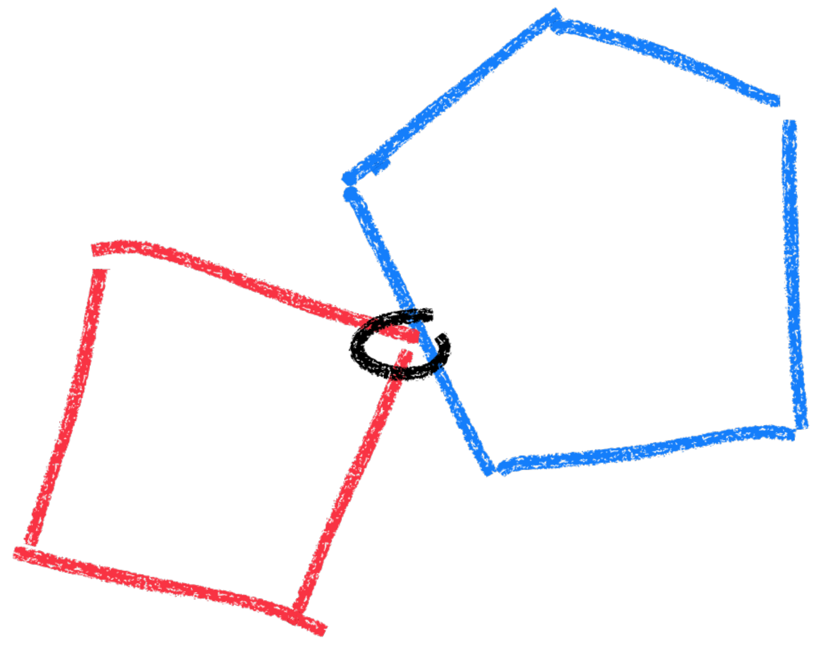
- do they collide in this time step?
- ...and if so, when and where?

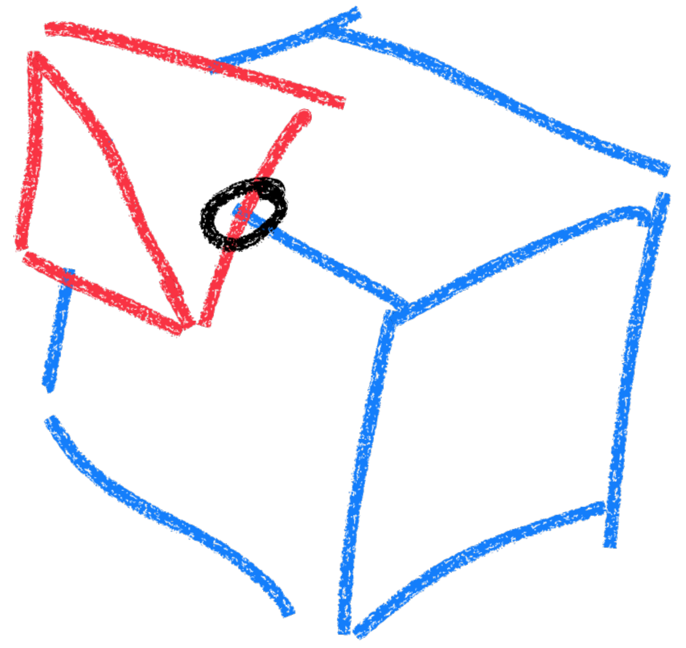


Common simplifications:

- limit to circles, spheres, triangles, line segments
- only allow for linear motion of vertices
- only consider non-degenerate cases
  - in 3D: vertex-face and edge-edge
  - in 2D: vertex-edge
- degenerate cases can be handled as an extreme case of one of these



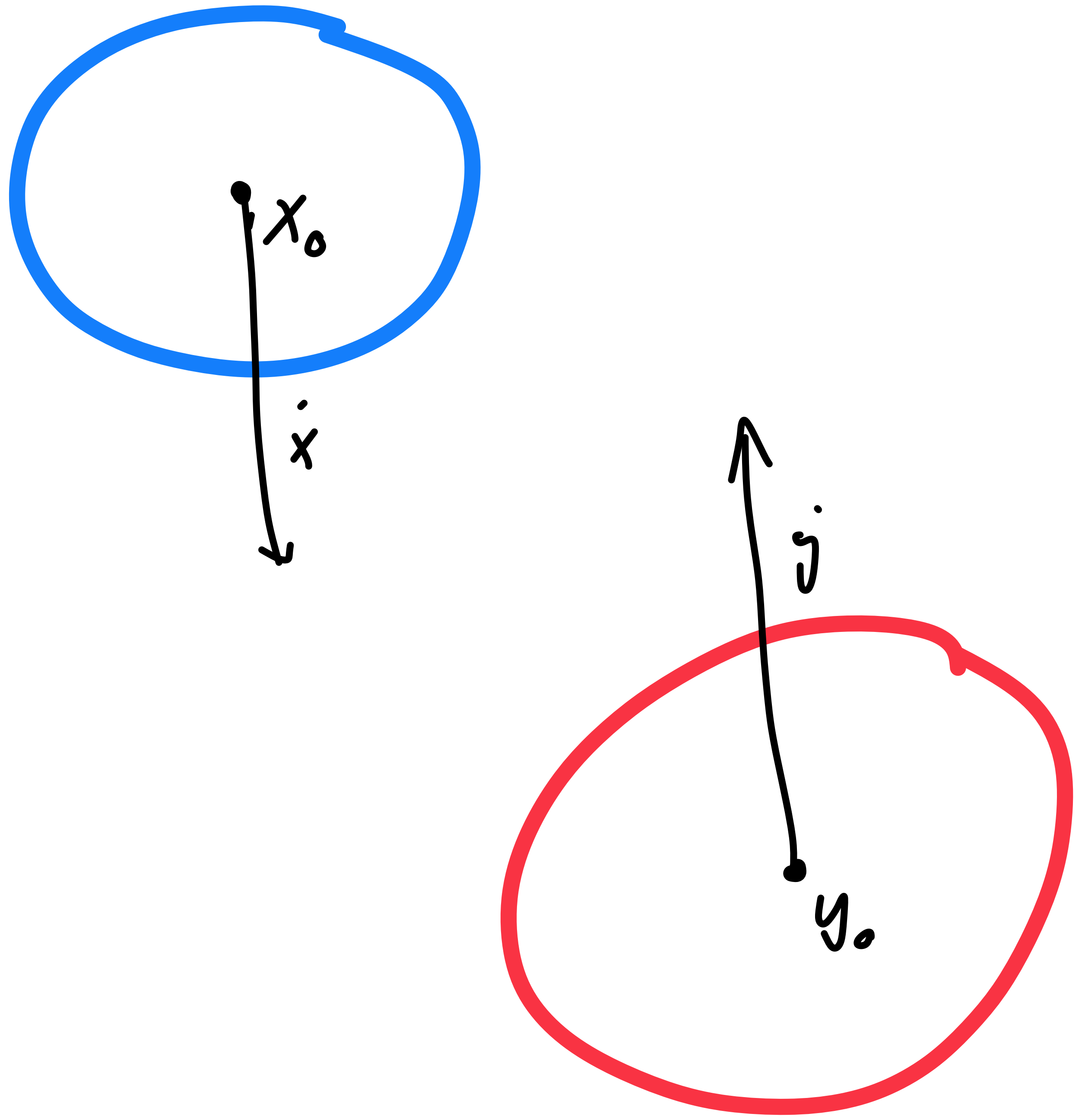




CCD for spheres

Given $\mathbf{x}_0, \dot{\mathbf{x}}, \mathbf{y}_0, \dot{\mathbf{y}}, r_x, r_y$

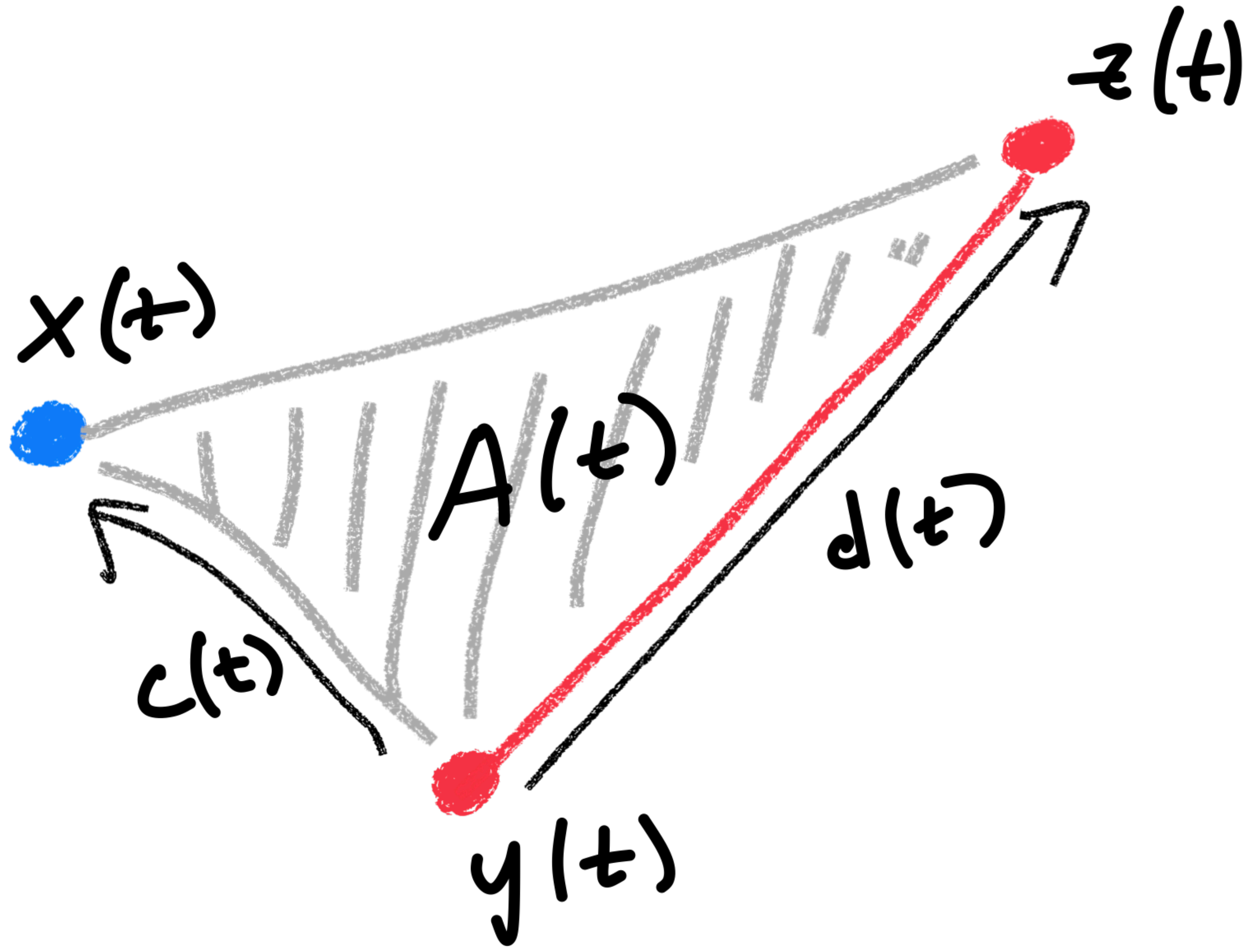
- is there a time $t \in (0,h]$ where the centers are at a distance $r_x + r_y$?
- positions are $\mathbf{x}(t) = \mathbf{x}_0 + t\dot{\mathbf{x}}$ and $\mathbf{y}(t) = \mathbf{y}_0 + t\dot{\mathbf{y}}$
- let $\mathbf{d}_0 = \mathbf{x}_0 - \mathbf{y}_0$; $\dot{\mathbf{d}} = \dot{\mathbf{x}} - \dot{\mathbf{y}}$; $R = r_x + r_y$
- difference is $\mathbf{d}(t) = \mathbf{d}_0 + t\dot{\mathbf{d}}$
- collision when $\|\mathbf{d}(t)\| = R$ or $(\mathbf{d}_0 + t\dot{\mathbf{d}}) \cdot (\mathbf{d}_0 + t\dot{\mathbf{d}}) = R^2$
- quadratic: $(\dot{\mathbf{d}} \cdot \dot{\mathbf{d}})t^2 + 2(\mathbf{d}_0 \cdot \dot{\mathbf{d}})t + (\mathbf{d}_0 \cdot \mathbf{d}_0 - R^2) = 0$
- there is a collision iff there is a root in $(0,h]$
- smallest root in $(0,h]$ is the collision time
- (déjà vu … remember ray-sphere intersection?)



CCD for line segments

The only nondegenerate case is vertex-edge

- vertex $\mathbf{x}(t)$ and edge endpoints $\mathbf{y}(t)$ and $\mathbf{z}(t)$
- given: $\mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0, \dot{\mathbf{x}}, \dot{\mathbf{y}}, \dot{\mathbf{z}}$
- collision occurs when $\{\mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t)\}$ are collinear and $\mathbf{x}$ is between $\mathbf{y}$ and $\mathbf{z}$
- simple collinearity test: area of triangle is zero
- triangle edges $\mathbf{c}(t) = \mathbf{x}(t) - \mathbf{y}(t) = \mathbf{c}_0 + t\dot{\mathbf{c}}$ and $\mathbf{d}(t) = \mathbf{z}(t) - \mathbf{y}(t) = \mathbf{d}_0 + t\dot{\mathbf{d}}$
- area $2A(t) = \mathbf{c}(t) \wedge \mathbf{d}(t)$, set to zero
- quadratic $(\dot{\mathbf{c}} \wedge \dot{\mathbf{d}})t^2 + (\mathbf{c}_0 \wedge \dot{\mathbf{d}} + \dot{\mathbf{c}} \wedge \mathbf{d}_0)t + (\mathbf{c}_0 \wedge \mathbf{d}_0) = 0$
- smallest root in $(0,h]$ for which $\mathbf{x}$ is between $\mathbf{y}$ and $\mathbf{z}$ (if any) is the collision time



$$
\begin{aligned}
\mathbf{v} \wedge \mathbf{w} &= (\mathbf{v} \times \mathbf{w})_z \\
&= v_x w_y - v_y w_x
\end{aligned}
$$

Robust quadratic formula

We all learned the quadratic formula in high school

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

What they didn't tell us

- there are two equally reasonable quadratic formulas

$$t = \frac{2c}{-b \mp \sqrt{b^2 - 4ac}}$$

- each one is inaccurate for certain cases (e.g. a or c near zero)
- if you just type in the familiar formula, you will sometimes get inaccurate collisions!

More stable procedure:

- compute $D = b^2 - 4ac$; if $D < 0$ there are no roots
- compute $r = -\frac{1}{2}\left(b + \mathrm{sign}(b)\sqrt{D}\right)$ (no subtraction, no cancellation!)
- roots are $t_1 = \frac{r}{a}$ and $t_2 = \frac{c}{r}$ (exercise: show that these are equal when $D = 0$)
- (see Numerical Recipes or other intro numerics textbooks)

CCD for triangle meshes

Here we have both edge-edge and point-face collisions

Analogous approach to 2D works

- both cases are actually the same (weird!)
- collision happens when the 4 involved vertices are coplanar, aka. volume of tetrahedron is zero
- points $\mathbf{w}(t), \mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t)$, velocities $\dot{\mathbf{w}}(t), \ldots, \dot{\mathbf{z}}(t)$
- think about tetrahedron edges $\mathbf{a} = \mathbf{x} - \mathbf{w}$, $\mathbf{b} = \mathbf{y} - \mathbf{w}$, $\mathbf{c} = \mathbf{z} - \mathbf{w}$
- $2V(t) = \det \begin{bmatrix} \mathbf{a}(t) & \mathbf{b}(t) & \mathbf{c}(t) \end{bmatrix} = \mathbf{a}(t) \cdot (\mathbf{b}(t) \times \mathbf{c}(t)) = 0$
- this is a cubic equation in t; collision time is the smallest root in $[0,h)$ for which the objects actually collide (vertex inside triangle, or line intersection inside edges)