Next few weeks

- Shading Models
  - Chapter 7
- Textures
- Graphics Pipeline
To compute images…

• Light Emission
  – What are the light sources?

• Light Propagation
  – Fog/Clear?

• Light Reflection
  – Interaction with media

Types of Lights

• Directional lights
  – E.g., sunlight
  – Light vector fixed direction

• Point lights
  – E.g., bulbs
  – Light position fixed
Types of Lights

- **Spot lights**: Like point light, but also
  - Cut-off angle
  - Attenuation

Types of Lights

- **Area Lights**: generate soft shadows
Types of Light

• Environment Maps

To compute images…

• Light Emission
  – What are the light sources?

• Light Propagation
  – Fog/Clear?

• Light Reflection
  – Interaction with media
Bidirectional Reflectance Distribution Function (BRDF)

Surface reflective characteristics

- **Spectral distribution**
  - Responsible for surface color
  - Tabulate in independent wavelength bands, or RGB

- **Spatial distribution**
  - Material properties vary with surface position
  - Texture maps

- **Directional distribution**
  - BRDF
  - Tabulation is impractical because of dimensionality
Radiometry

- Radiometry: measurement of light energy

- Defines relation between
  - Power
  - Energy
  - Radiance
  - Radiosity

Radiometric Terms

- Power: energy per unit time

- Irradiance: Incident power per unit surface area
  - From all directions
  - Watt/m²

- Radiosity: Exitant power per unit surface area
  - Same units
Radiance

• Radiance is radiant energy at x in direction $\theta$: 5D function
  – Power
    ▪ per unit projected surface area
    ▪ per unit solid angle
  – units: Watt / m$^2$.sr

Why is radiance important?

• Response of a sensor (camera, human eye) is proportional to radiance

• Pixel values in image proportional to radiance received from that direction
Materials - Three Forms

- Ideal diffuse (Lambertian)
- Ideal specular
- Directional diffuse

Reflectance—Three Forms

- Ideal diffuse (Lambertian)
- Directional diffuse
- Ideal specular
Ideal Diffuse Reflection

- Characteristic of multiple scattering materials
- An idealization but reasonable for matte surfaces
- Basis of most radiosity methods

Ideal Diffuse

- Lambert’s Law

\[ I_{\text{diffuse}} = I_{\text{light}} k_d \cos(\theta) \]
\[ I_{\text{diffuse}} = I_{\text{light}} k_d N \cdot L \]
Ideal Specular Reflection

- Calculated from Fresnel’s equations
- Exact for polished surfaces
- Basis of early ray-tracing methods

Directional Diffuse Reflection

- Characteristic of most rough surfaces
- Described by the BRDF
Classes of Models for the BRDF

• Plausible simple functions
  – Phong 1975;

• Physics-based models
  – Cook/Torrance, 1981; He et al. 1992;

• Empirically-based models
  – Ward 1992

Phong Shading Model

• Classic Phong
  – Ambient
  – Diffuse
  – Specular (Phong highlight)
  – Also fog and transparency possible

• For each light evaluate above
Specular

- Specular
  - Simulates surface smoothness
  - $(\max \{N \cdot H, 0\})^{shininess}$
  - $H = \frac{L + V}{|L + V|}$

Phong Reflection Model

- $Diffuse = k_d(N \cdot L)$
- $Specular = k_s(R \cdot V)^n$
The Blinn-Phong Model

\[ \text{Diffuse} = k_d (N \cdot L) \]
\[ \text{Specular} = k_s (N \cdot H)^n \]

Phong Shading Model

- \( I = \text{ambient} + \text{diffuse} + \text{specular} \)
  \[ I = k_a I_a + k_d I_d (N \cdot L) + k_s I_s (N \cdot H)^n \]

- We want all the \( I \)'s and \( k \)'s to be functions of (R,G,B)
  - \( I \)'s are function of light
  - \( k \)'s are function of material

- Sum over all lights
Terms in Phong

• Ambient
  – “Fake” global illumination
  – Fixed from all directions
    ▪ Makes it not black
The Phong Model

- Computationally simple
- Visually pleasing

Phong: Reality Check

Real photographs

Phong model
Phong: Reality Check

• Doesn’t represent physical reality
  – Energy not conserved
  – Not reciprocal
  – Maximum always in specular direction

Reciprocity

• Interchange L and V
  – Photon doesn’t know its direction
  – Same behavior

• Blinn-Phong vs. Phong
Classes of Models for the BRDF

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  – Ward 1992, Lafortune model

Motivation for Cook-Torrance

• Plastic has substrate that is white with embedded pigment particles
  – Colored diffuse component
  – White specular component

• Metal
  – Specular component depends on metal
  – Negligible diffuse component
Cook-Torrance BRDF Model

- Phong: too smooth
- A *microfacet* model
  - Surface modeled as random collection of planar facets
  - Incoming ray hits exactly one facet, at random
Facet Reflection

- Input: probability distribution of facet angle
- $H$ vector used to define facets that contribute

Cook-Torrance BRDF Model

- “Specular” term (really directional diffuse)
Facet distribution

Facet Distribution

- \( D \) function describes distribution of \( H \)
- Formula due to Beckmann
  - Statistical model
  - Alpha is angle between \( N \) and \( H \)
    - Intuitively, deviation of microgeometry from macro normal
  - \( m \) is RMS slope of microfacets: large \( m \) means more spread out reflections

\[
D = \frac{1}{4m^2 \cos^4 \alpha} e^{-\left(\frac{\tan \alpha}{m}\right)^2}
\]
Cook-Torrance BRDF Model

Masking/shadowing
Masking and Shadowing

\[ G = \min \left[ 1, \frac{2N_i H_i N_i V}{V.H}, \frac{2N_i H_i N_i L}{V.H} \right] \]

Fresnel Reflection Properties

- Gives coefficients when light moves between different media
- Polarization
- Captures behavior of metals and dielectrics
- Explains why reflection increases (and surfaces appear more “mirror”-like) at grazing angles
Metal vs. Nonmetal

Fresnel reflectance

Metals

Nonmetals (k=0)

Highly Non-Linear

$R_F$

angle of incidence $\theta_i$

- copper
- aluminum
- iron
- diamond
- glass
- water
Fresnel Equations

\[ \eta_1 \sin(\theta_1) = \eta_2 \sin(\theta_2) \]

\[ F_p = \frac{\eta_2 \cos(\theta_1) - \eta_1 \cos(\theta_2)}{\eta_2 \cos(\theta_1) + \eta_1 \cos(\theta_2)} \]
\[ F_s = \frac{\eta_1 \cos(\theta_1) - \eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1) + \eta_2 \cos(\theta_2)} \]

Fresnel Reflectance

\[ F = \frac{F_s + F_p}{2} \]

for unpolarized light

- Equations apply for metals and nonmetals
  - for metals, use complex index : \( n + ik \)
  - for nonmetals/dielectrics, \( k = 0 \)
Schlick’s approximation of Fresnel

\[ R_F(\theta) = R_F(0) + (1 - R_F(0))(1 - \cos(\theta))^5 \]

- For dielectric

\[ R_F(0) = \left(\frac{\eta - 1}{\eta + 1}\right)^2 \]
### $R_F(0)$

<table>
<thead>
<tr>
<th>Insulator:</th>
<th>Volumetric Reflectance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>0.02, 0.02, 0.02</td>
</tr>
<tr>
<td>Plastic</td>
<td>0.03, 0.03, 0.03</td>
</tr>
<tr>
<td>Glass</td>
<td>0.08, 0.08, 0.08</td>
</tr>
<tr>
<td>Diamond</td>
<td>0.17, 0.17, 0.17</td>
</tr>
<tr>
<td>Gold</td>
<td>1.00, 0.71, 0.29</td>
</tr>
<tr>
<td>Silver</td>
<td>0.95, 0.93, 0.88</td>
</tr>
<tr>
<td>Copper</td>
<td>0.95, 0.64, 0.54</td>
</tr>
<tr>
<td>Iron</td>
<td>0.56, 0.57, 0.58</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.91, 0.92, 0.92</td>
</tr>
</tbody>
</table>

### Rob Cook’s vases

![Rob Cook’s vases](image)

*Source: Cook, Torrance 1981*
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Measured BRDFs
Ward Model

- Physically valid
  - Energy conserving
  - Satisfies reciprocity
  - Easy to integrate

- Based on empirical data

Ward Model

- Isotropic and anisotropic materials
Ward Model: Isotropic

\[ f_s = \rho_s \frac{1}{4\pi m^2} \frac{1}{\sqrt{N.L.N.V}} e^{-\frac{\tan^2 \theta_H}{m^2}} \]

• where,
  – \( m \) (usually \( \alpha \)) is surface roughness

Ward Model: Anisotropic

\[ f_s = \rho_s \frac{1}{4\pi m_x m_y} \frac{1}{\sqrt{N.L.N.V}} e^{-\frac{\tan^2 \theta_H (\frac{\cos^2 \phi_H}{m_x^2} + \frac{\sin^2 \phi_H}{m_y^2})}{m^2}} \]

\[ f_s = \rho_s \frac{1}{4\pi m_x m_y} \frac{1}{\sqrt{N.L.N.V}} e^{-\frac{\left(\frac{H_x}{m_x}\right)^2 + \left(\frac{H_y}{m_y}\right)^2}{1+N.H}} \]

• where,
  – \( m_x, m_y \) are surface roughness in \( \hat{x}, \hat{y} \)
  – \( \hat{x}, \hat{y} \) are mutually perpendicular to the normal

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Examples

Teapot
Normals for Illumination

- In polygonal models, each facet has normal
- But, faceted look (N constant)
  - Directional light (constant diffuse illumination)

Shading Normals

- Normal matches the object (not the polygons)
  - Assume polygons are piecewise smooth approximation
  - Ideally provided by underlying object
  - Otherwise, average normals of nearby facets
Shading Models

- Fast, easy: Phong
- Physically-based model: Cook-Torrance
- Empirically-based model: Ward

- Next time: textures

Books

- Email about RTR (3rd ed.)