Next few weeks

- Shading Models
  - Chapter 7
- Textures
- Graphics Pipeline
To compute images...

• Light Emission
  – What are the light sources?

• Light Propagation
  – Fog/Clear?

• Light Reflection
  – Interaction with media

Types of Lights

• Directional lights
  – E.g., sunlight
  – Light vector fixed direction

• Point lights
  – E.g., bulbs
  – Light position fixed
Types of Lights

• Spot lights: Like point light, but also
  – Cut-off angle
  – Attenuation

Types of Lights

• Area Lights: generate soft shadows
Types of Light

• Environment Maps

To compute images…

• Light Emission
  – What are the light sources?

• Light Propagation
  – Fog/Clear?

• Light Reflection
  – Interaction with media
Surface reflective characteristics

- **Spectral distribution**
  - Responsible for surface color
  - Tabulate in independent wavelength bands, or RGB

- **Spatial distribution**
  - Material properties vary with surface position
  - Texture maps

- **Directional distribution**
  - BRDF
  - Tabulation is impractical because of dimensionality
Radiometry

• Radiometry: measurement of light energy

• Defines relation between
  – Power
  – Energy
  – Radiance
  – Radiosity

Radiometric Terms

• Power: energy per unit time

• Irradiance: Incident power per unit surface area
  – From all directions
  – Watt/m²

• Radiosity: Exitant power per unit surface area
  – Same units
Radiance

- Radiance is radiant energy at x in direction $\theta$: 5D function
  - Power
    - per unit projected surface area
    - per unit solid angle
  - units: Watt / m$^2$.sr

Why is radiance important?

- Response of a sensor (camera, human eye) is proportional to radiance
- Pixel values in image proportional to radiance received from that direction
Materials - Three Forms

Ideal diffuse (Lambertian)

Ideal specular

Directional diffuse

Reflectance - Three Forms

Ideal diffuse
(Lambertian)

Directional
diffuse

Ideal specular
Ideal Diffuse Reflection

- Characteristic of multiple scattering materials
- An idealization but reasonable for matte surfaces
- Basis of most radiosity methods

Ideal Diffuse

- Lambert’s Law

\[ I_{diffuse} = I_{light} k_d \cos(\theta) \]
\[ I_{diffuse} = I_{light} k_d N \cdot L \]
Ideal Specular Reflection

• Calculated from Fresnel’s equations
• Exact for polished surfaces
• Basis of early ray-tracing methods

Directional Diffuse Reflection

• Characteristic of most rough surfaces
• Described by the BRDF
Classes of Models for the BRDF

- Plausible simple functions
  - Phong 1975;

- Physics-based models
  - Cook/Torrance, 1981; He et al. 1992;

- Empirically-based models
  - Ward 1992

Phong Shading Model

- Classic Phong
  - Ambient
  - Diffuse
  - Specular (Phong highlight)
  - Also fog and transparency possible

- For each light evaluate above
Specular

- Specular
  - Simulates surface smoothness
  - $\max \{N \cdot H, 0\}^{\text{shininess}}$
  - $H = \frac{L + V}{||L + V||}$

Phong Reflection Model

$\text{Diffuse} = k_d(N \cdot L)$
$\text{Specular} = k_s(R \cdot V)^n$
The Blinn-Phong Model

Half-Vector

L

H

V

Specular

Diffuse = \( k_d(N \cdot L) \)

Specular = \( k_s(N \cdot H)^n \)

Phong Shading Model

- \( I = \) ambient + diffuse + specular

\[ I = k_a I_a + k_d I_d(N \cdot L) + k_s I_s(N \cdot H)^n \]

- We want all the \( I \)'s and \( k \)'s to be functions of \( (R,G,B) \)
  - \( I \)'s are function of light
  - \( k \)'s are function of material

- Sum over all lights
Terms in Phong

- Ambient
  - “Fake” global illumination
  - Fixed from all directions
    - Makes it not black
The Phong Model

- Computationally simple
- Visually pleasing

Phong: Reality Check

Real photographs

Phong model
**Phong: Reality Check**

- Doesn’t represent physical reality
  - Energy not conserved
  - Not reciprocal
  - Maximum always in specular direction

**Reciprocity**

- Interchange L and V
  - Photon doesn’t know its direction
  - Same behavior

- Blinn-Phong vs. Phong
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  - Ward 1992, Lafortune model

Motivation for Cook-Torrance

- Plastic has substrate that is white with embedded pigment particles
  - Colored diffuse component
  - White specular component

- Metal
  - Specular component depends on metal
  - Negligible diffuse component
Cook-Torrance BRDF Model

- Phong: too smooth
- A *microfacet* model
  - Surface modeled as random collection of planar facets
  - Incoming ray hits exactly one facet, at random
Facet Reflection

- Input: probability distribution of facet angle
- $H$ vector used to define facets that contribute

Cook-Torrance BRDF Model

- “Specular” term (really directional diffuse)

$$f_s = \rho_s \frac{F_{DG}}{\pi N.L N.V}$$
Facet Distribution

- $D$ function describes distribution of $H$
- Formula due to Beckmann
  - Statistical model
  - Alpha is angle between N and H
    - Intuitively, deviation of microgeometry from macro normal
  - $m$ is RMS slope of microfacets: large $m$ means more spread out reflections

$$D = \frac{1}{4m^2 \cos^4 \alpha} e^{-\left[\tan \frac{\alpha}{m}\right]^2}$$
Cook-Torrance BRDF Model

Images by Rob Cook, Program of Computer Graphics, Cornell University

© Kavita Bala, Computer Science, Cornell University
Fresnel Reflection Properties

- Gives coefficients when light moves between different media
- Polarization
- Captures behavior of metals and dielectrics
- Explains why reflection increases (and surfaces appear more “mirror”-like) at grazing angles

\[ G = \min\left[1, \frac{2N_H N_V}{V_H}, \frac{2N_H N_L}{V_H}\right] \]
Metal vs. Nonmetal

Fresnel reflectance

Metals

Nonmetals (k=0)

Highly Non-Linear

angle of incidence $\theta_i$

$R_F$

copper  aluminum  iron  diamond  glass  water
Fresnel Equations

\[ \eta_1 \sin(\theta_1) = \eta_2 \sin(\theta_2) \]

\[ F_p = \frac{\eta_2 \cos(\theta_1) - \eta_1 \cos(\theta_2)}{\eta_2 \cos(\theta_1) + \eta_1 \cos(\theta_2)} \]

\[ F_s = \frac{\eta_1 \cos(\theta_1) - \eta_2 \cos(\theta_2)}{\eta_1 \cos(\theta_1) + \eta_2 \cos(\theta_2)} \]

Fresnel Reflectance

\[ F = \frac{F_s + F_p}{2} \]

for unpolarized light

- Equations apply for metals and nonmetals
  - for metals, use complex index : \( n + ik \)
  - for nonmetals/dielectrics, \( k = 0 \)
Schlick’s approximation of Fresnel

\[ R_F(\theta) = R_F(0) + (1 - R_F(0))(1 - \cos(\theta))^5 \]

• For dielectric

\[ R_F(0) = \left( \frac{\eta - 1}{\eta + 1} \right)^2 \]
### $R_F(0)$

<table>
<thead>
<tr>
<th>Insulator: Water</th>
<th>0.02, 0.02, 0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulator: Plastic</td>
<td>0.03, 0.03, 0.03</td>
</tr>
<tr>
<td>Insulator: Glass</td>
<td>0.08, 0.08, 0.08</td>
</tr>
<tr>
<td>Insulator: Diamond</td>
<td>0.17, 0.17, 0.17</td>
</tr>
<tr>
<td>Metal: Gold</td>
<td>1.00, 0.71, 0.29</td>
</tr>
<tr>
<td>Metal: Silver</td>
<td>0.95, 0.93, 0.88</td>
</tr>
<tr>
<td>Metal: Copper</td>
<td>0.95, 0.64, 0.54</td>
</tr>
<tr>
<td>Metal: Iron</td>
<td>0.56, 0.57, 0.58</td>
</tr>
<tr>
<td>Metal: Aluminum</td>
<td>0.91, 0.92, 0.92</td>
</tr>
</tbody>
</table>

### Rob Cook’s vases

- Carbon
- Red Rubber
- Obsidian
- Lunar Dust
- Olive Drab
- Rust
- Bronze
- Tungsten
- Copper
- Tin
- Nickel
- Stainless Steel

Source: Cook, Torrance 1981
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Measured BRDFs

- White paint
- Blue paint
- Commercial aluminum
- Blue plastic
Ward Model

• Physically valid
  – Energy conserving
  – Satisfies reciprocity
  – Easy to integrate

• Based on empirical data

Ward Model

• Isotropic and anisotropic materials
Ward Model: Isotropic

\[ f_s = \rho_s \frac{1}{4\pi m^2} \frac{1}{\sqrt{N.LN.V}} e^{-\frac{\tan^2 \theta h}{m^2}} \]

- where,
  - \( m \) (usually \( \alpha \)) is surface roughness

Ward Model: Anisotropic

\[ f_s = \rho_s \frac{1}{4\pi m_x m_y \sqrt{N.LN.V}} e^{-\tan^2 \theta h \left( \frac{\cos^2 \phi h}{m_x^2} + \frac{\sin^2 \phi h}{m_y^2} \right)} \]

\[ f_s = \rho_s \frac{1}{4\pi m_x m_y \sqrt{N.LN.V}} e^{-\frac{(\frac{H_x}{m_x})^2 + (\frac{H_y}{m_y})^2}{1 + N.H}} \]

- where,
  - \( m_x, m_y \) are surface roughness in \( \hat{x}, \hat{y} \)
  - \( \hat{x}, \hat{y} \) are mutually perpendicular to the normal
Examples

Teapot
**Normals for Illumination**

- In polygonal models, each facet has a normal.
- But, faceted look (N constant)
  - Directional light (constant diffuse illumination)

**Shading Normals**

- Normal matches the object (not the polygons)
  - Assume polygons are piecewise smooth approximation
  - Ideally provided by underlying object
  - Otherwise, average normals of nearby facets
Shading Models

• Fast, easy: Phong
• Physically-based model: Cook-Torrance
• Empirically-based model: Ward

• Next time: textures

Books

• Email about RTR (3rd ed.)