CS5540: Computational Techniques for Analyzing Clinical Data

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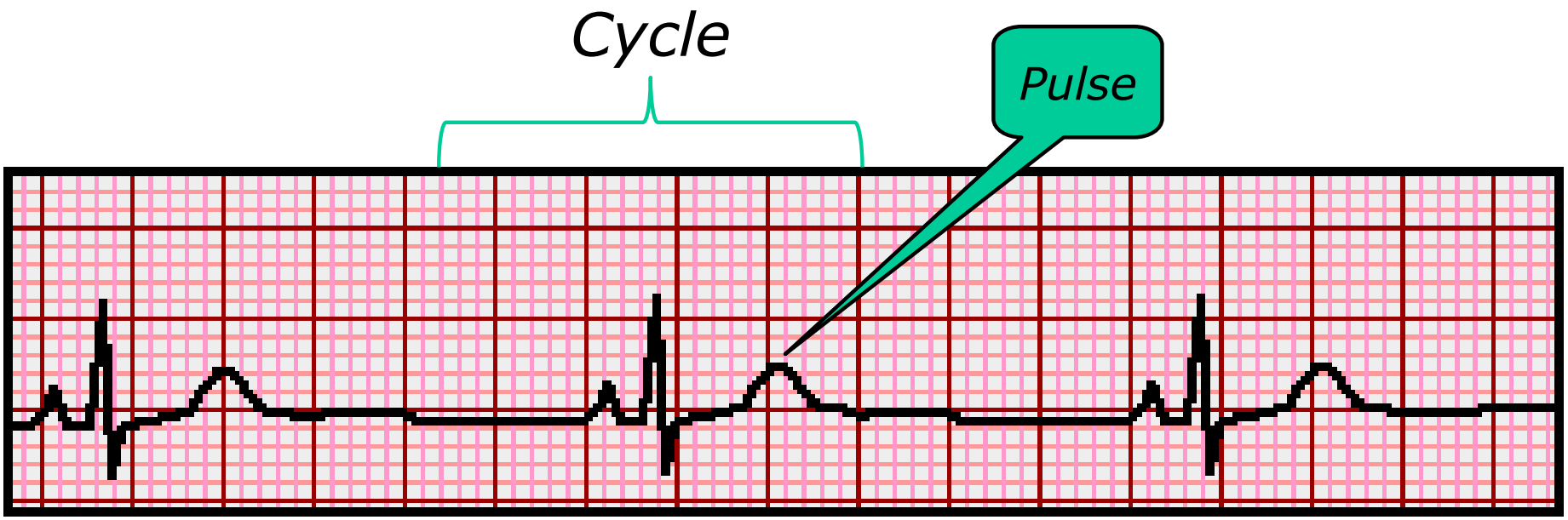
Cycles and pulses



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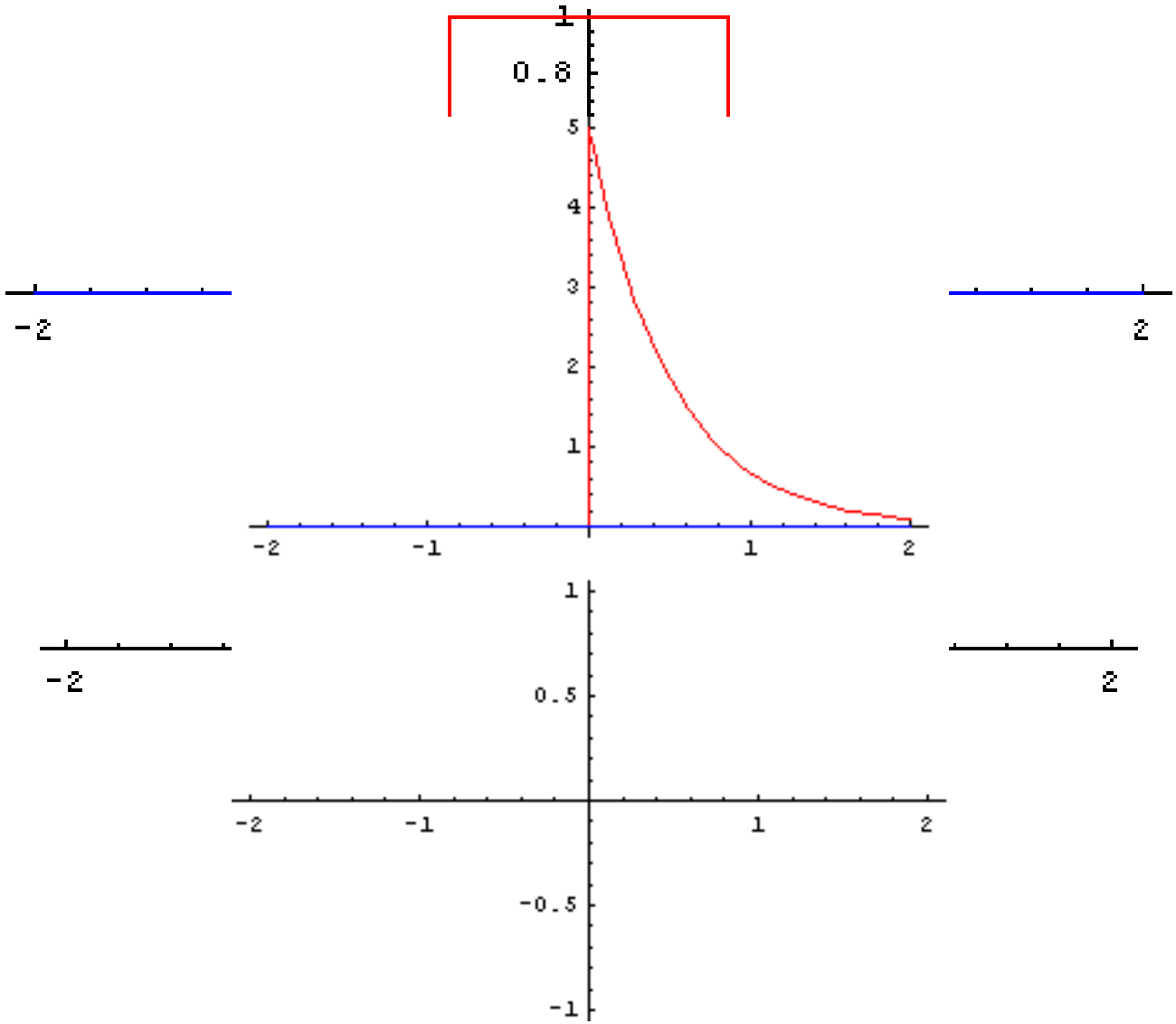
Note: <u>cycle</u> is standard terminology, but <u>pulse</u> is not. The ECG literature calls these "waves" but we want to be more general than just ECG.

Convolution in action



Convolution as a dot product

- Note: technically this is correlation
  - Same as convolution when symmetric

[… 19 20 -4 17 93 …]

[1 -1] = 19 – 20 = -1

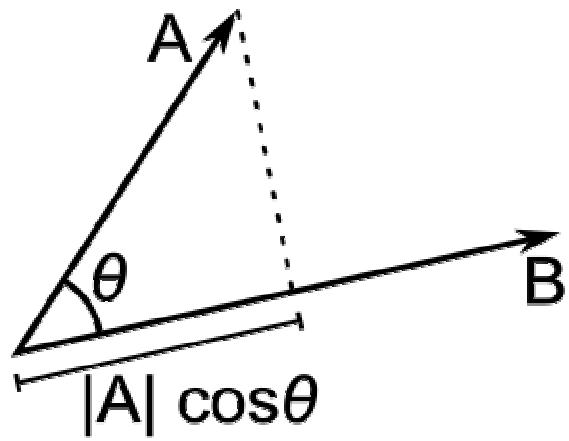
[1 -1] = 20 – (-4) = 24

[1 -1] = -4 – 17 = -21

- This seems like a lot of mechanism to get somewhere totally obvious

Geometric intuition

- Taking the dot product of two vectors
  - Recall [a b c]@[e f g] = ae + bf + cg
  - Basically the projection of a vector on another



- The normalized vector with the biggest projection on x is, of course: x!
  - Useful intuition for transforms, etc.

Normalization

- Matched filter must be normalized
  - Want to find our template in the ECG data
- What looks like [1 2 6]?
  - Obviously, [… 1 2 6 …] does (score = 41)
  - What about [… 150 100 50…]?
  - Subtract mean from both template and ECG
    - Ex: [-1 1] template
  - Also need to normalize the "energy" to 1
    - Sum of squares

Convolution and edge detection

- Strategy: smooth (convolve with a Gaussian) then find zero crossings
- We can also look for inflection points (2nd derivative is zero) after smoothing
- Or we can look for big changes in the ECG
  - Sort of what we did with the matched filter
- Can do smoothing at the same time!

Convolution's properties

- Convolution is also popular because it is a surprisingly deep topic
  - With lots of connections to other areas
    - Central limit theorem
    - Multiplication of polynomials
    - Etc, etc.

Gaussian convolution

- The standard way to smooth data is to convolve with a Gaussian
  - Related to Gaussian noise assumption
- Gaussian noise assumption itself comes from one of the Gaussian's many amazing properties, the Central Limit Theorem

Random variables

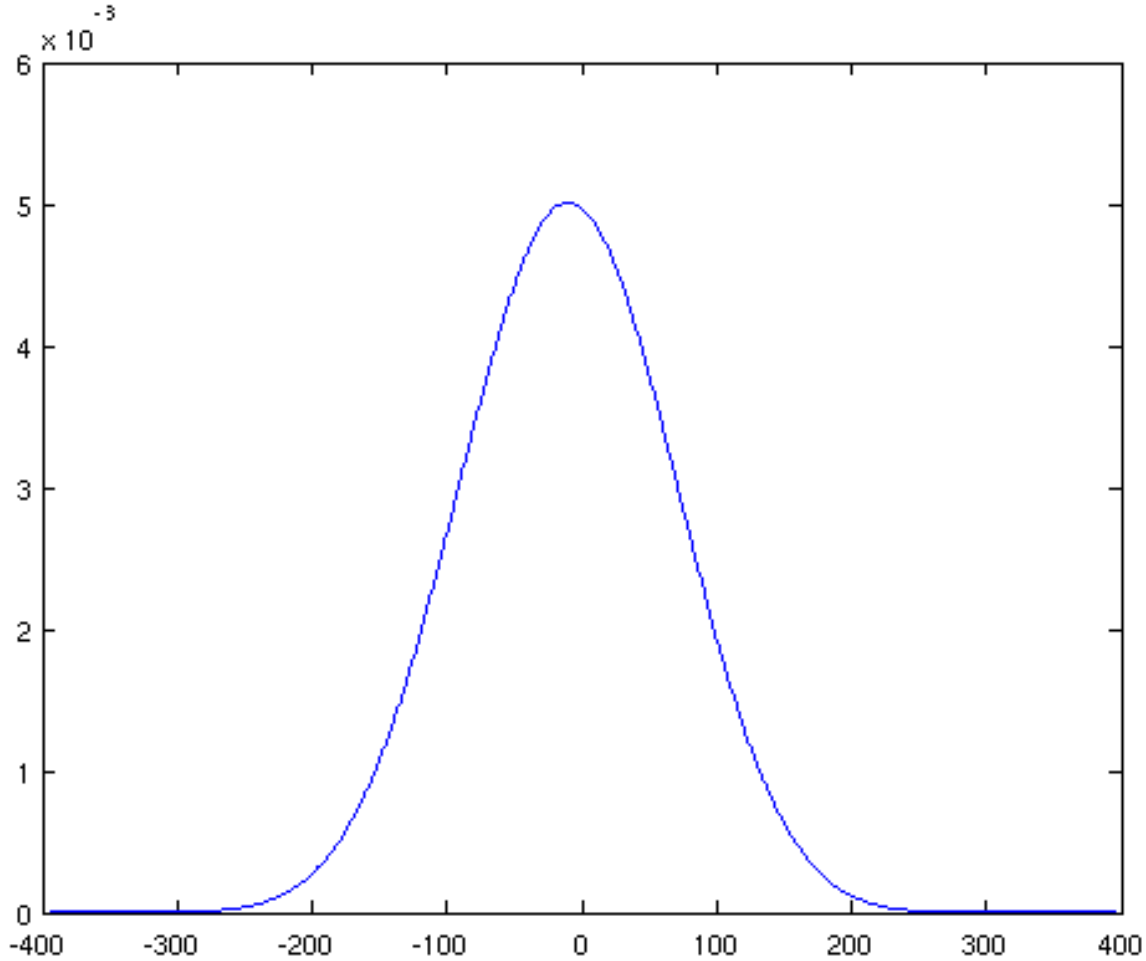
- A discrete random variable assigns each integer a probability
  - Ex: a perfect coin or a die
  - The assignment function is the density
- What happens when we toss a two dice and add them up?
  - This is another random variable
  - Q: What is the density?
  - A: Convolution of the dice density with itself

Limit theorems

- Let's throw more and more dice, adding up their totals
  - Ex: 1000 dice
    - Sum is from 1000 to 6000
  - If the dice density is uniform, we'll typically get around 3500
    - What about an arbitrary density?
- Central limit theorem: we will end up with a Gaussian

Illustration

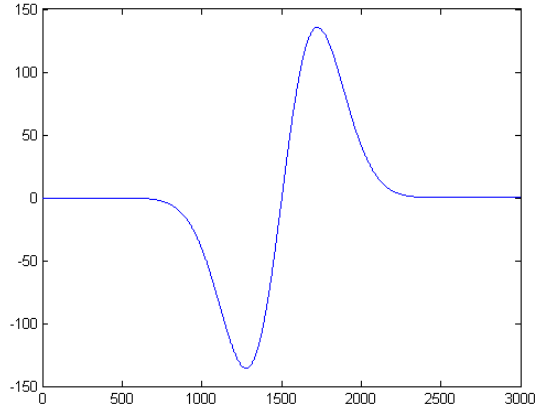
- How fast does this become Gaussian?

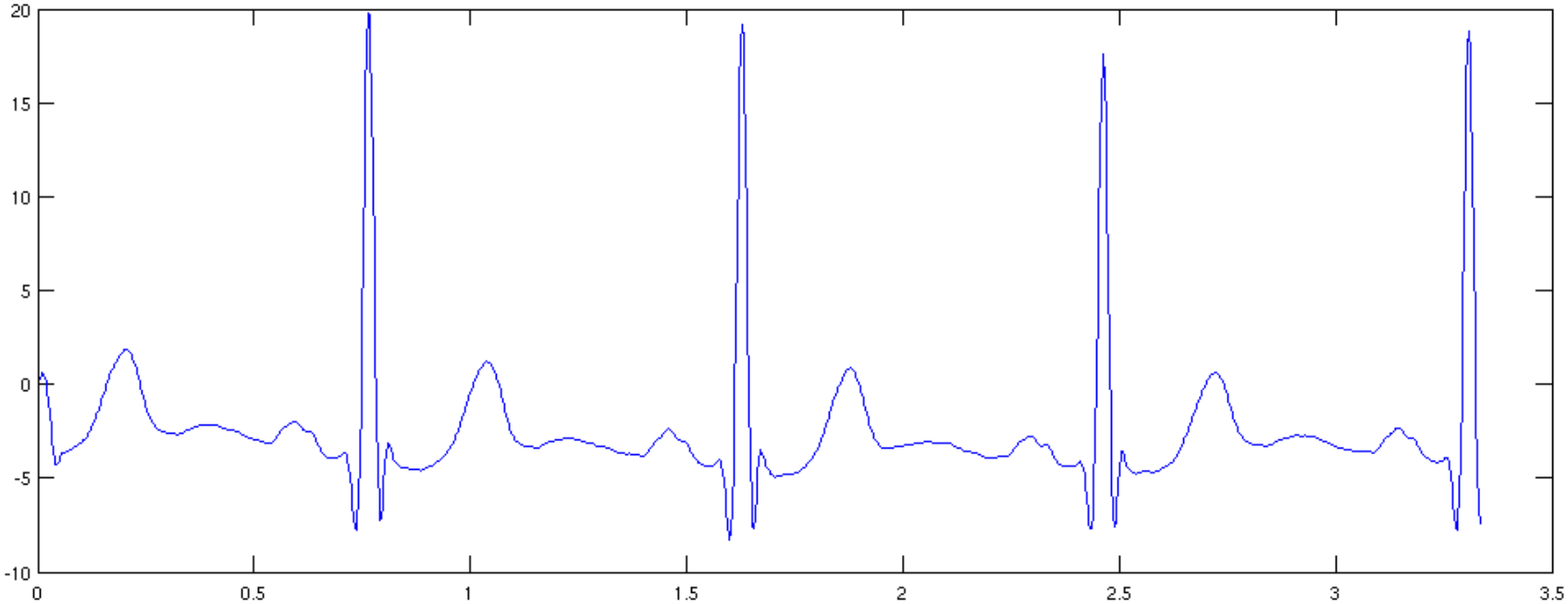


Convolution and edges

- We want to smooth then find zero crossings or big increases/decreases
- There is some nice math that tells us we can do this in a single step
  - It was actually illustrated in the matched filter

Pulse finding example





Linearity

- Scalars don't matter
  - Scalars in Y, not T (the latter is scale invariance, which is much harder)
- Addition doesn't matter
- Formally, for any two ECG's X,Y and scalars a,b we have

$$H(aX + bY) = a\,H(x) + b\,H(y)$$

- Many devices/systems are linear when they are well behaved

Shift invariance

- Shift the input, shift the output
- Or: starting point doesn't matter
- Note that fixed shift itself is a convolution

LSI systems

- Many important systems have both of these properties
  - Linear Shift Invariant
  - Aka Linear Time Invariant

LSI and convolution

- Convolution is an LSI operation
  - Pretty obvious (think about local averaging)
- But any LSI is actually convolution with something, which is easy to find
  - Why would this be true??

Unit impulse response

- Unit impulse: [… 0 1 0 …]
- Any input is the sum of scaled and shifted unit impulses
  - Example: to get [1 6 -10] we:
    - Take the unit impulse shifted once left: [1 0 0]
    - Add 6 times the unit impulse, no shift: [1 6 0]
    - Subtract 10 times the unit impulse, shifted right