Weill Cornell Medical College

IMAGING DATA EVALUATION AND ANALYTICS LAB (IDEAL)

CS5540: Computational Techniques for Analyzing Clinical Data

Lecture 15:

Accelerated MRI Image Reconstruction

Ashish Raj, PhD

Image Data Evaluation and Analytics Laboratory (IDEAL)

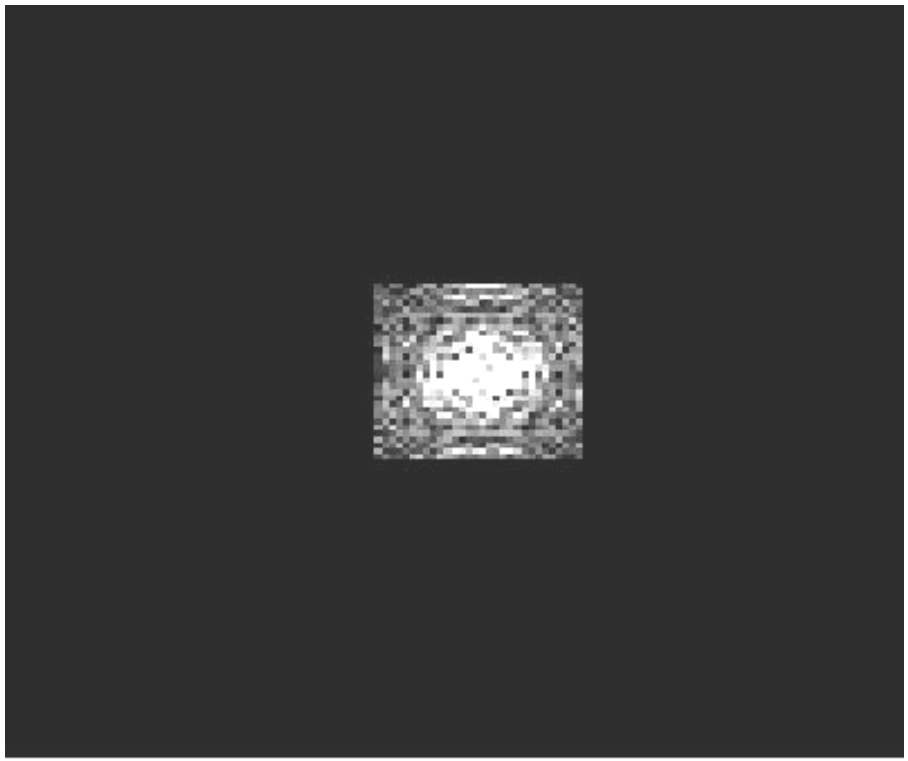
Department of Radiology

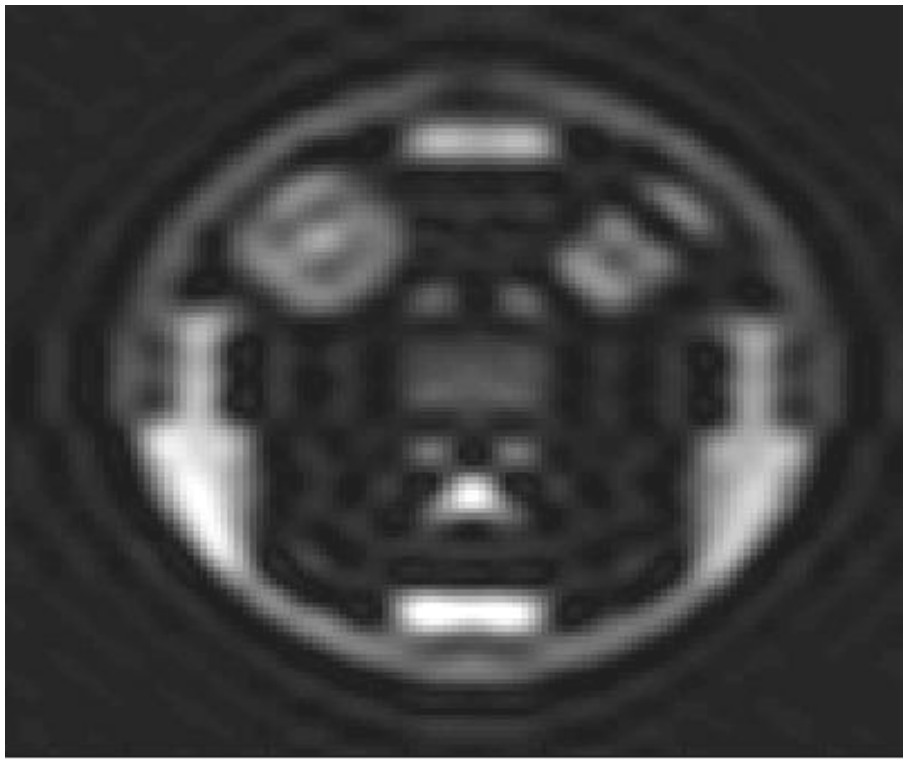
Weill Cornell Medical College

New York

Truncation

- Can make MRI faster by sampling less of k-space
- Truncation = sampling central part of k-space

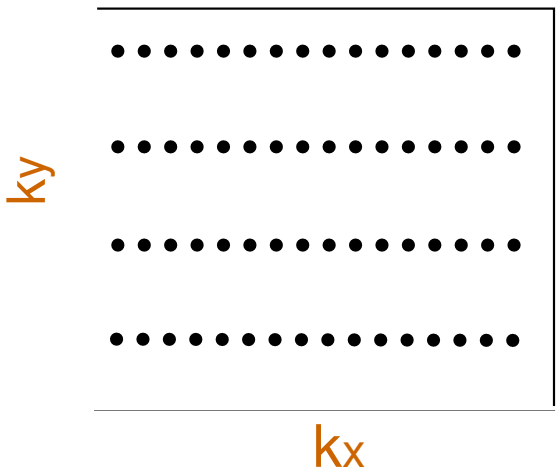




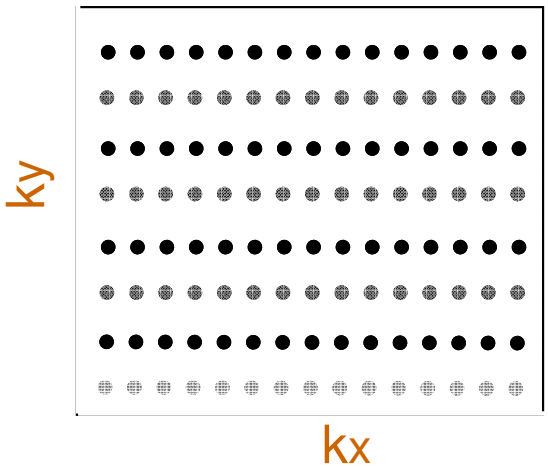
- Recon = super-resolution (extrapolating k-space)
- Very hard, no successful method yet

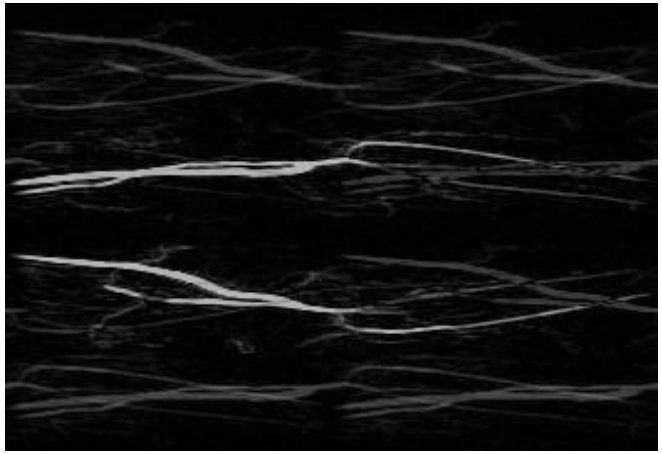
Parallel imaging

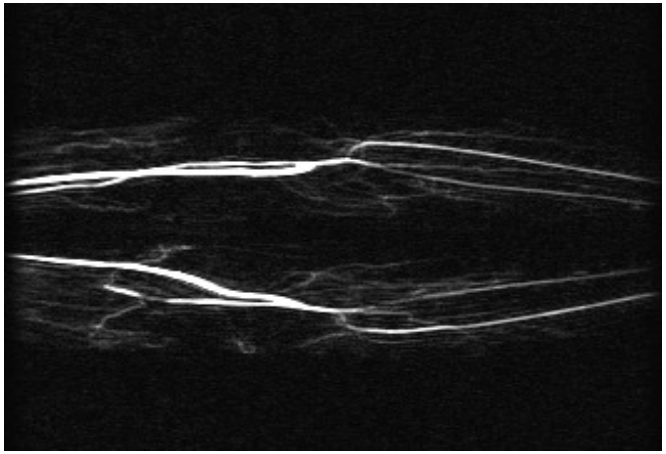
Under-sampled k-space



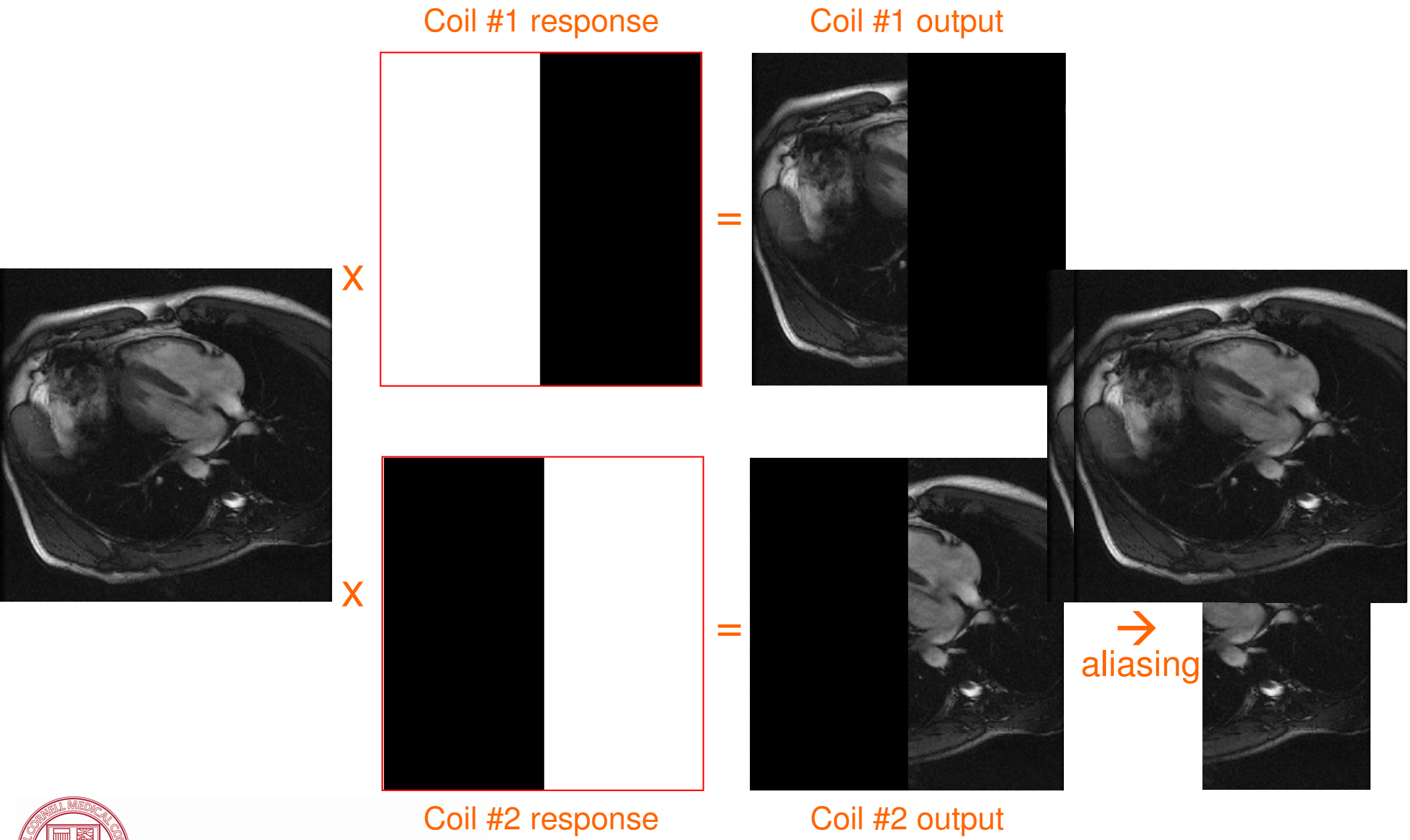
Reconstructed k-space



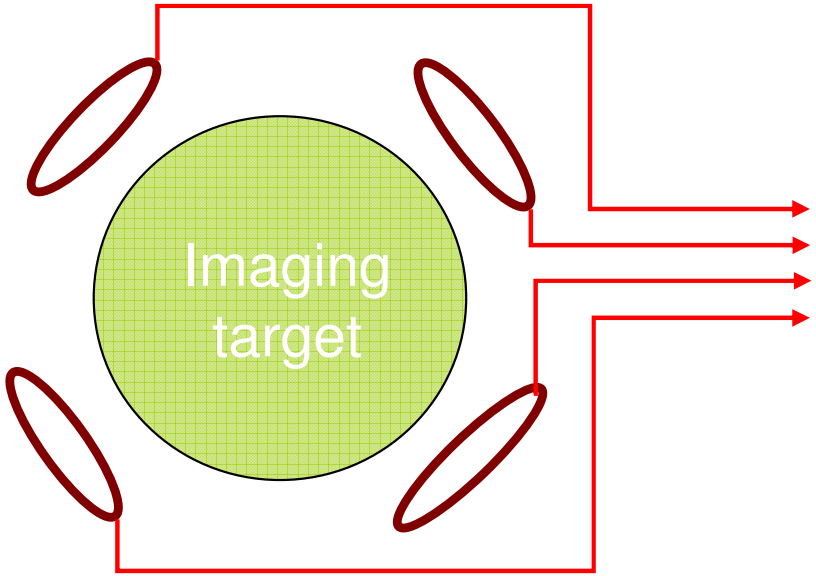


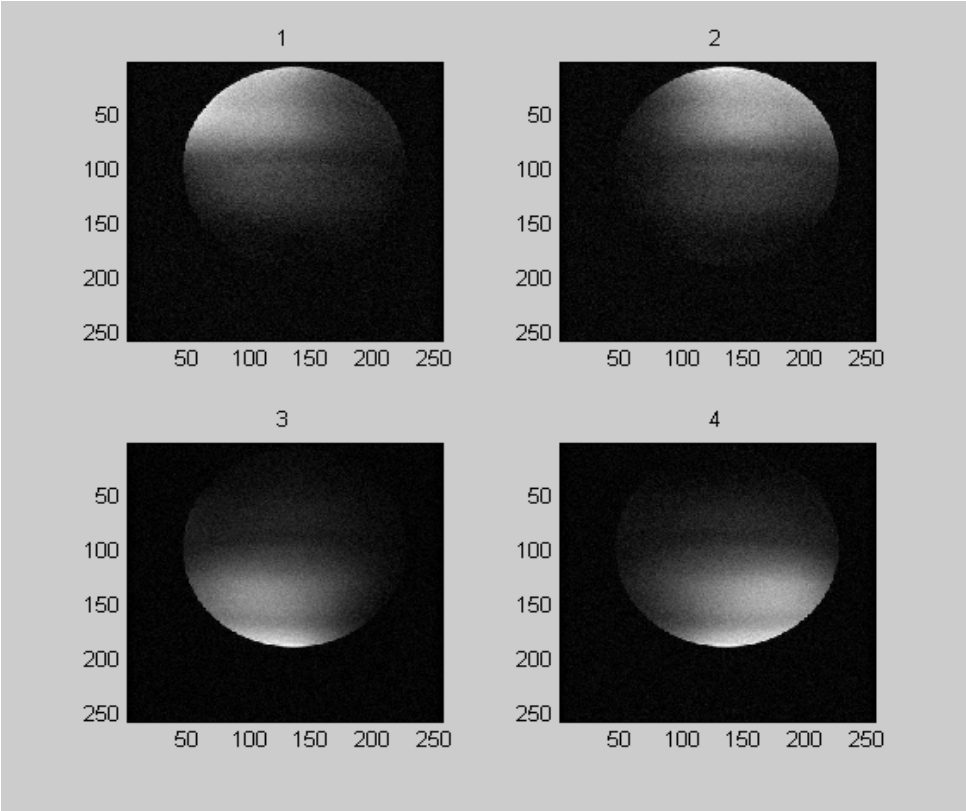


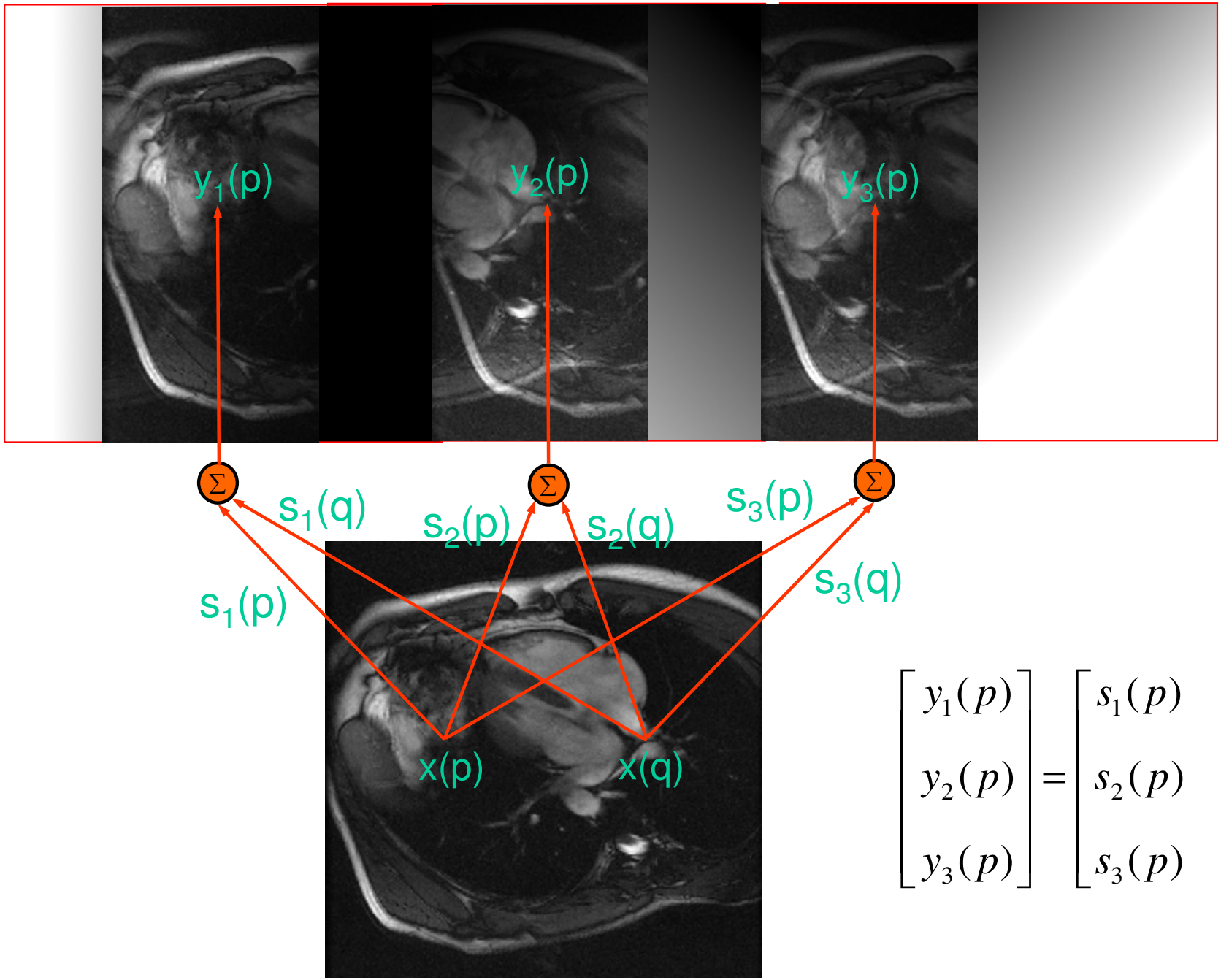
Different coil responses help



Parallel imaging with real coils







$$\begin{bmatrix} y_1(p) \\ y_2(p) \\ y_3(p) \end{bmatrix} = \begin{bmatrix} s_1(p) & s_1(q) \\ s_2(p) & s_2(q) \\ s_3(p) & s_3(q) \end{bmatrix} \begin{bmatrix} x(p) \\ x(q) \end{bmatrix}$$

Least squares solution

- Least squares estimate:

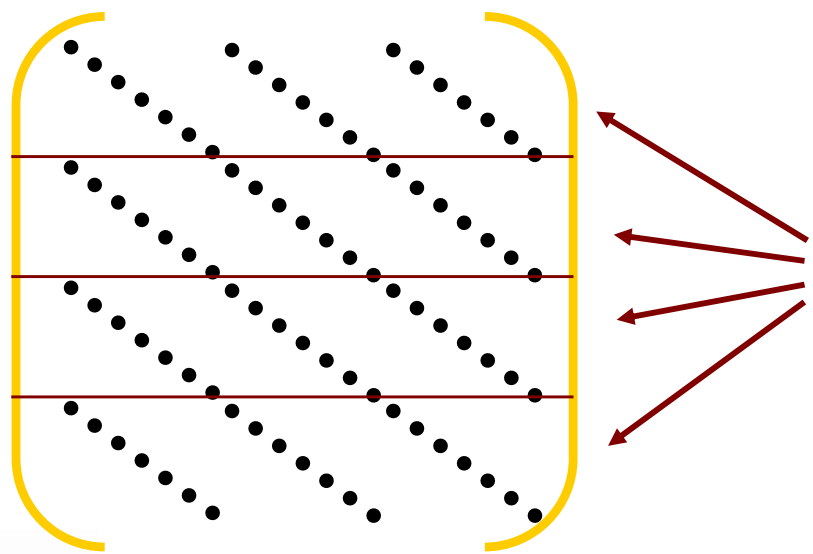
$$(\hat{x}(p), \hat{x}(q)) = \arg \min_{x(p),x(q)} \sum_{l \in \text{Coils}} \left[y_l(p) - s_l(p)x(p) - s_l(q)x(q) \right]^2$$

  – Famous MR algorithm: SENSE (1999)

- Linear inverse system

$$y = Hx + n$$

$$H =$$



Encodes different Coil outputs

Maximum a Posteriori (Bayesian) Estimate

- Consider the class of linear systems $y = Hx + n$
- Bayesian methods maximize the posterior probability:

$$Pr(x|y) \square Pr(y|x) \,.\, Pr(x)$$

- $Pr(y|x)$ (likelihood function) $= \exp(- ||y-Hx||^2)$
- $Pr(x)$ (prior PDF) $= \exp(-G(x))$
- Non-Bayesian: maximize only likelihood

$$x_{est} = arg\ min\ ||y-Hx||^2$$

- Bayesian:

$$x_{est} = arg\ min\ ||y-Hx||^2 + G(x)\ ,$$

where $G(x)$ is obtained from the prior distribution of x

- If $G(x) = ||Gx||^2$ ➔ *Tikhonov Regularization*

Maximum a Posteriori (Bayesian) Estimate = Energy Minimization

- Since parallel imaging is ill-posed, we need a stabilizing term

$$\hat{x} = \arg\min_x E(x) \equiv \left[\|y - Hx\|^2 + \lambda G(x)\right]$$

Makes Hx close to y

Makes x smooth or piecewise smooth

- This has a nice Bayesian interpretation
- Likelihood of x, assuming iid Gaussian noise, is

$$\Pr(y|x) \propto \exp(-\|y - Hx\|^2)$$

  – Write an arbitrary prior distribution on x as

$$\Pr(x) \propto \exp(-G(x))$$

Then we get above energy minimization!

Questions: What should G be? How do we minimize E?

Correct Prior Model Depends on Imaging Situation

- Temporal priors: smooth time-trajectory
- Sparse priors: L0, L1, L2 (=Tikhonov)
- Spatial Priors: most powerful for static images
- For static images we recommend robust spatial priors using Edge-Preserving Priors (EPPs)
- For dynamic images, we can use smoothness and/or sparsity in x-f space

Weill Cornell Medical College

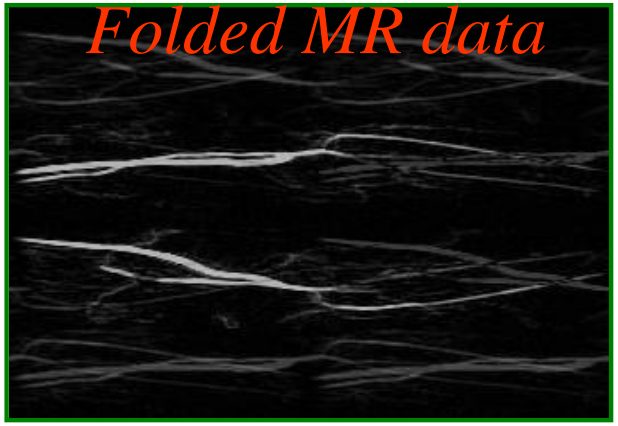
IMAGING DATA EVALUATION AND ANALYTICS LAB (IDEAL)

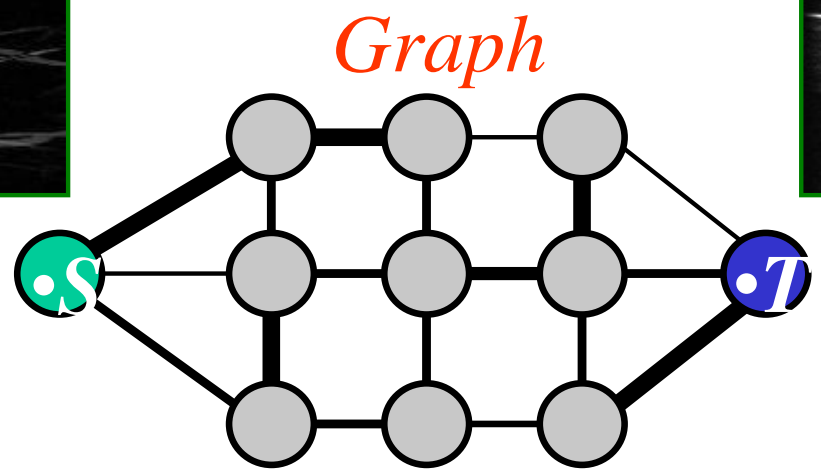
EPIGRAM: Edge-preserving Parallel Imaging Using Graph Cut Minimization

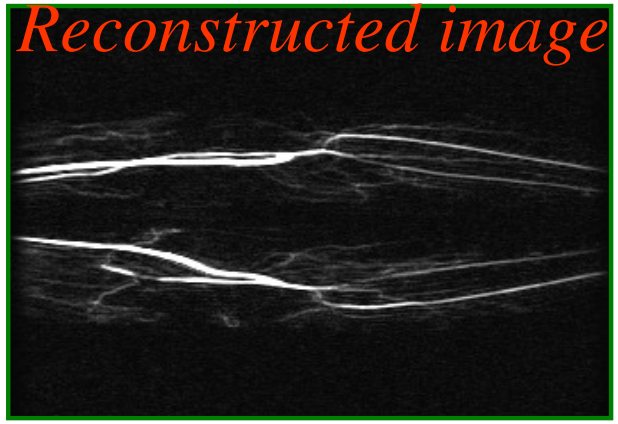
Joint work with: Ramin Zabih, Gurmeet Singh

MRI Reconstruction Using Graph Cuts

- *A new graph-based algorithm **
- *Inspired by advanced robotic vision, computer science*







- *Operations on this graph produce reconstructed image!*

- *Raj et al, Magnetic Resonance in Medicine, Jan 2007,*
- *Raj et al, Computer Vision and Pattern Recognition, 2006*

Use Edge-preserving Spatial Penalty

- Finds the MAP estimate

$$\hat{x} = \arg\min_{x} E(x) \equiv \left[\|y - Hx\|^2 + \lambda G(x)\right]$$

Makes Hx close to y

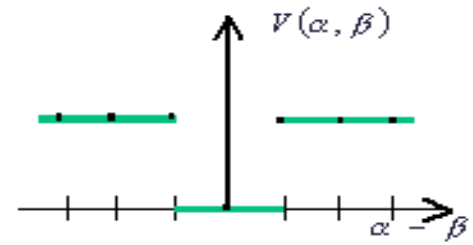
Makes x piecewise smooth

- Used Markov Random Field priors

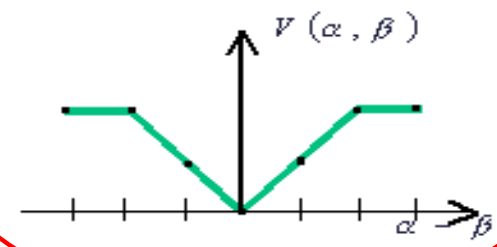
$$G(x) = \sum_{(p,q)\in\mathcal{N}} V(x_p - x_q)$$

–If V "levels off", this preserves edges

Potts function

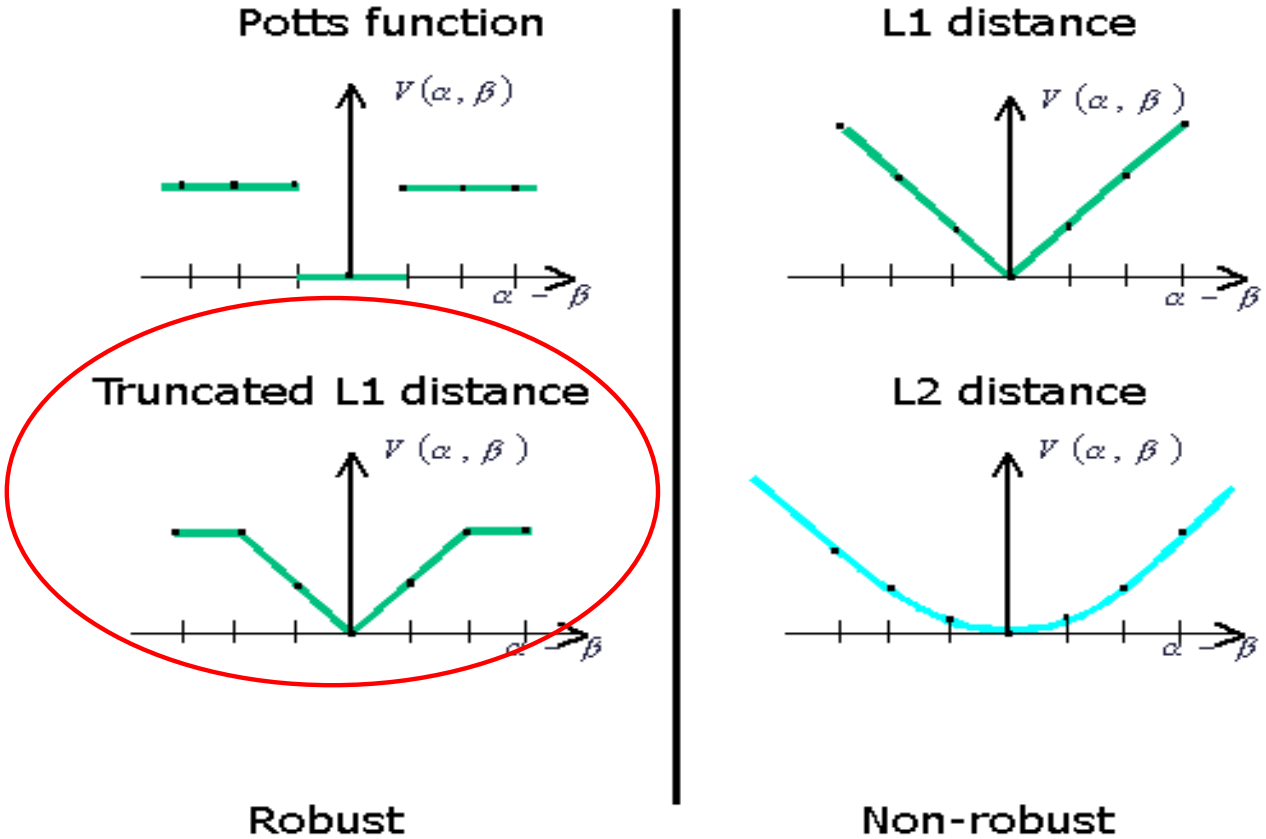


Truncated L1 distance



Robust

Examples of distance metrics

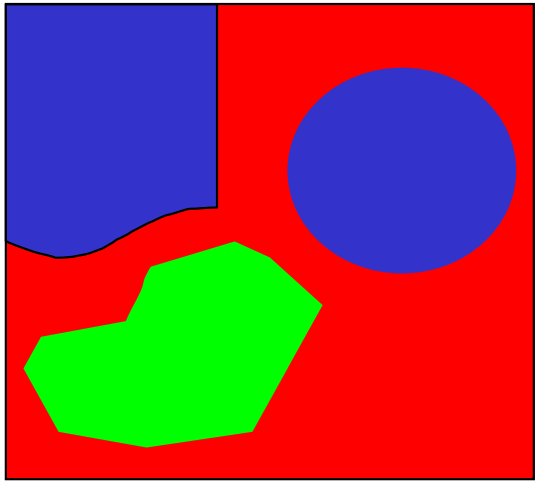


- Discontinuous, non-convex metric
- This is very hard for traditional minimization algorithms

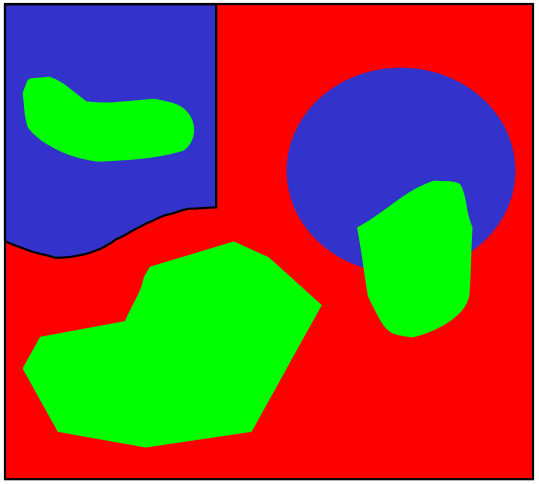
Expansion move algorithm

Original minimization problem is turned into a series of discrete (binary) problems, called EXPANSION MOVES

Input labeling f



Green expansion move from f



- Find green expansion move that most decreases E
  - Move there, then find the best blue expansion move, etc
  - Done when no α-expansion move decreases the energy, for any label α

Graph Cut minimization

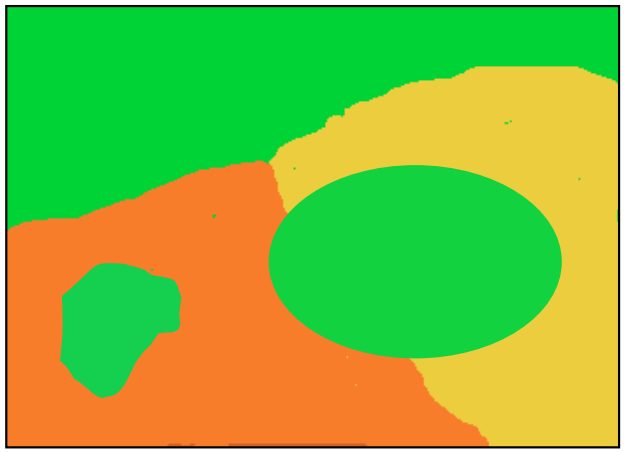
- Used Graph Cut technique – a combinatorial optimization method which can solve for non-convex energy functions
- Original minimization problem is turned into a series of discrete (binary) problems, called EXPANSION MOVES
- Each expansion move is a binary energy minimization problem, call it B(b)
- This binary problem is solved by graph cut

• Builds a graph whose nodes are image pixels, and whose edges have weights obtained from the energy terms in B(b)
• Minimization of B(b) is reduced to finding the minimum cut of this graph

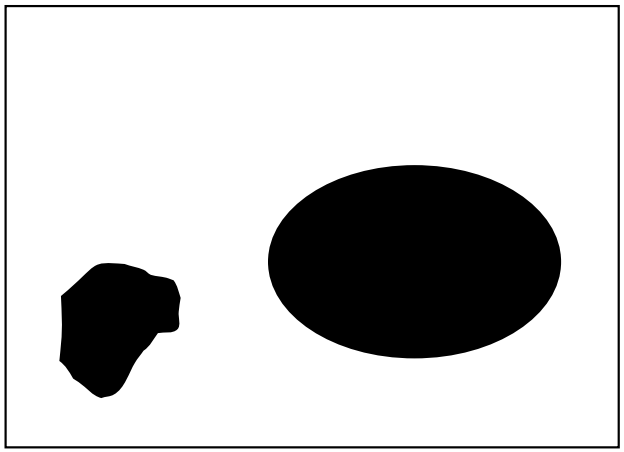
•Raj A, Singh G, Zabih R, Kressler B, Wang Y, Schuff N, Weiner M. Bayesian Parallel Imaging With Edge-Preserving Priors. Magn Reson Med. 2007 Jan;57(1):8-21

Binary sub-problem

Input labeling

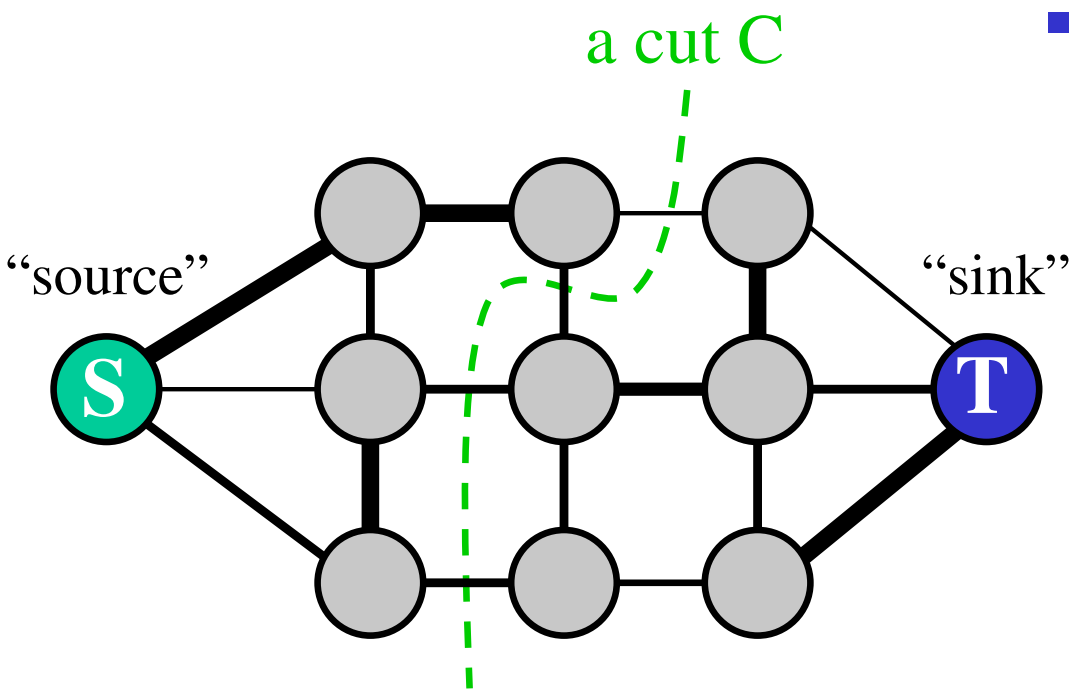


Expansion move



Binary image

Minimum cut problem

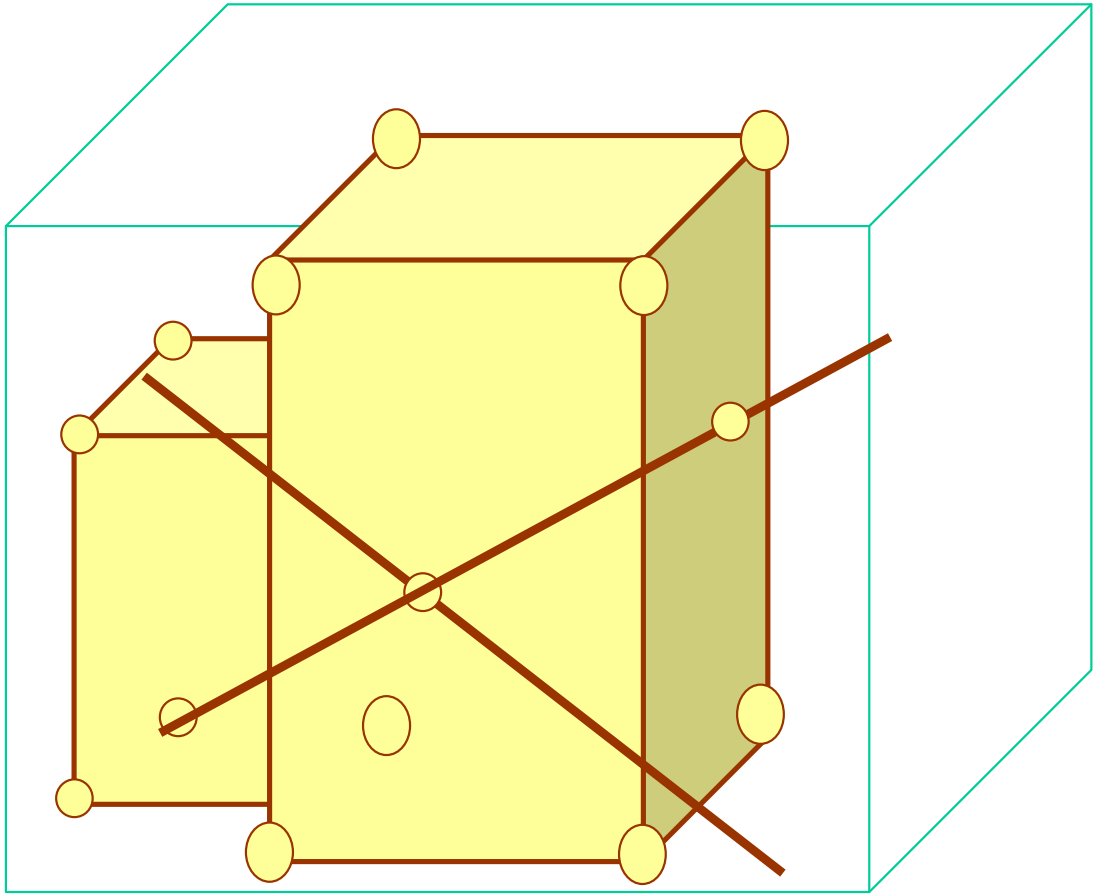


A graph with two terminals

- Min cut problem:
  - Find the cheapest way to cut the edges so that the "source" is separated from the "sink"
  - Cut edges going from source side to sink side
  - Edge weights now represent cutting "costs"

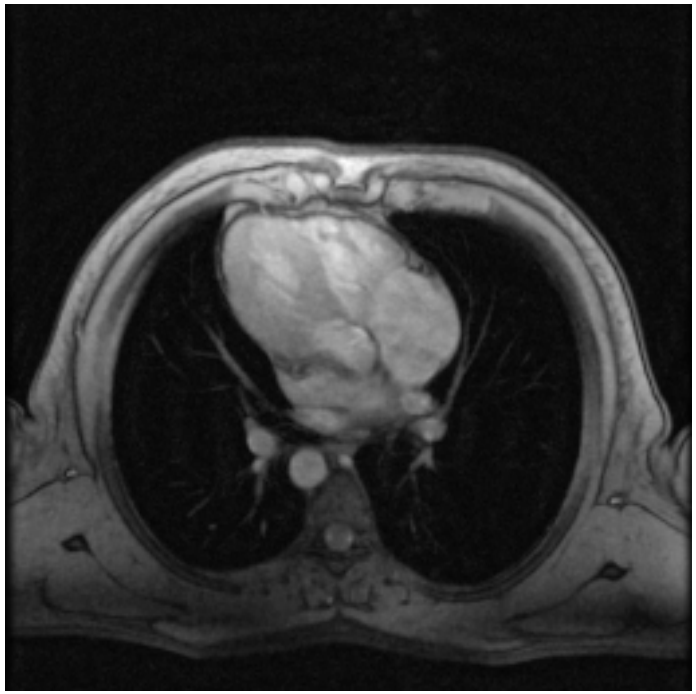
Mincut = binary assignment (source=0, sink=1)

Line Search vs Graph Cut Minimization



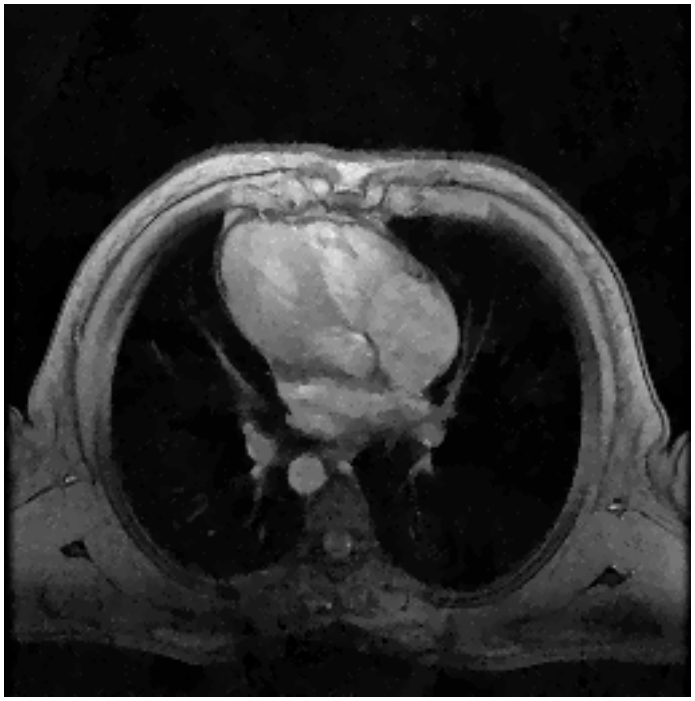
- Line search vs. global min over 2^n candidate points
- Vertices of an n-hypercube
- Global minima over candidate points, regardless of convexity and local minima!

In vivo data

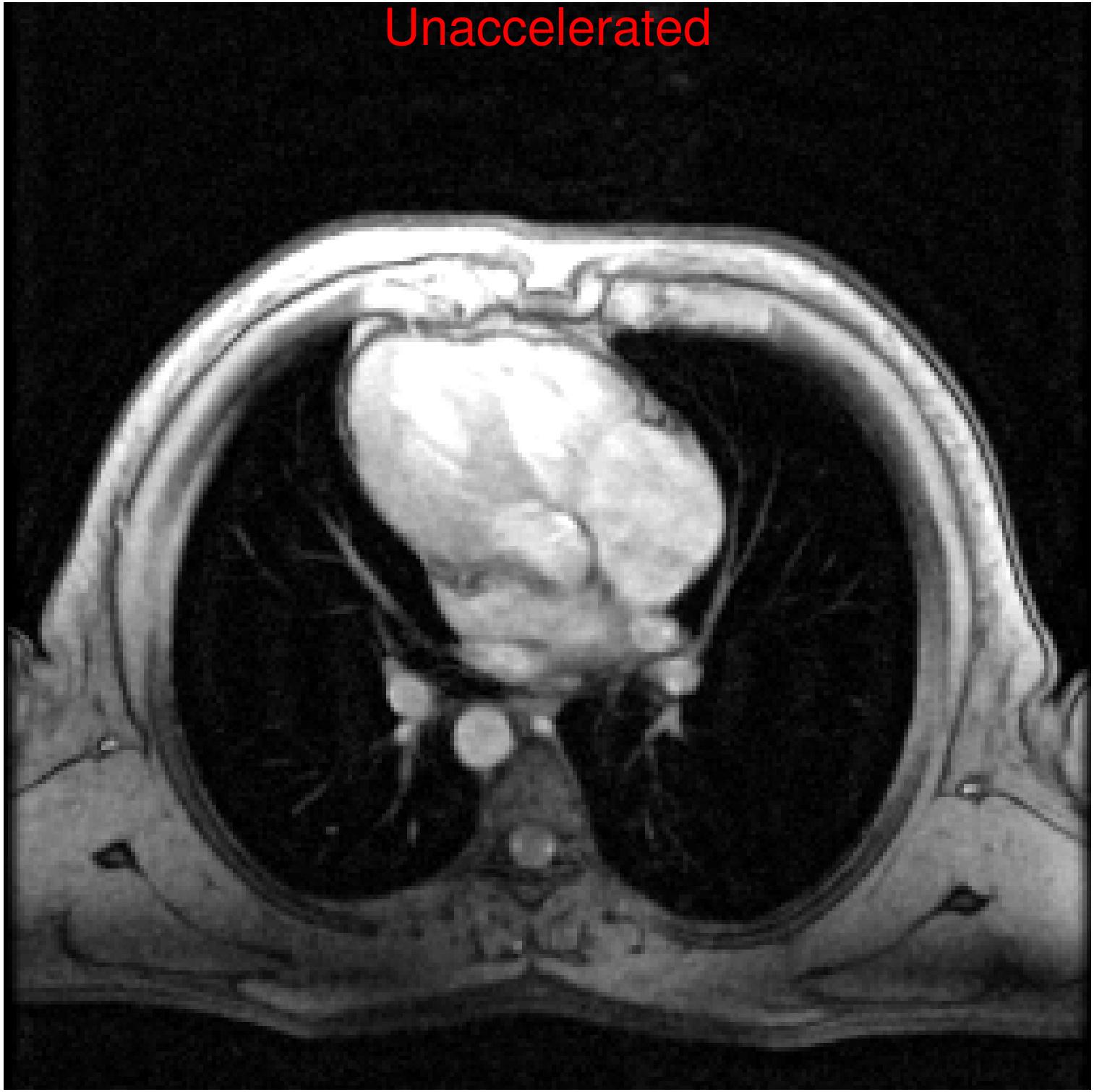


Unaccelerated image

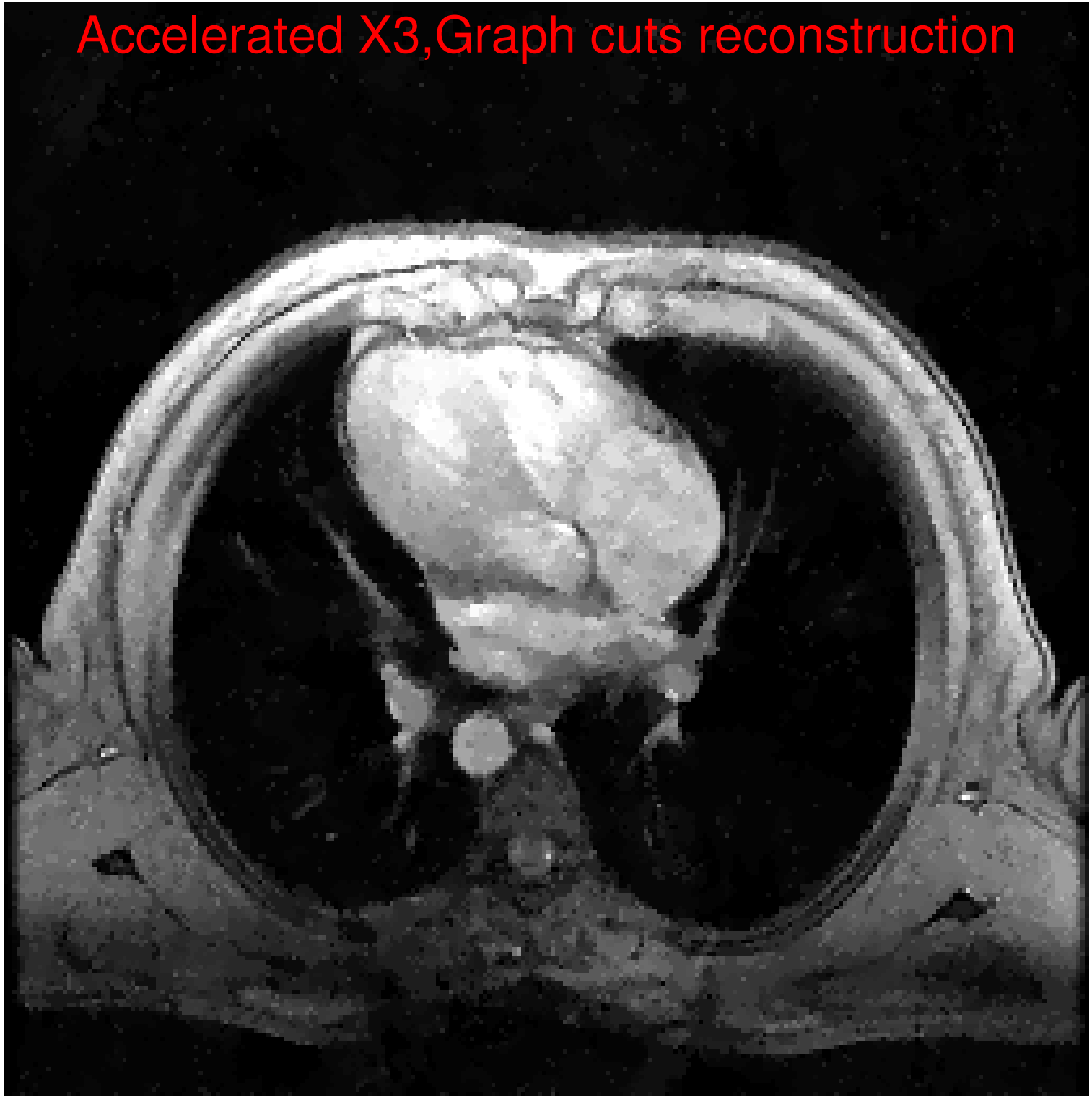
Accelerated X3,
SENSE reconstruction

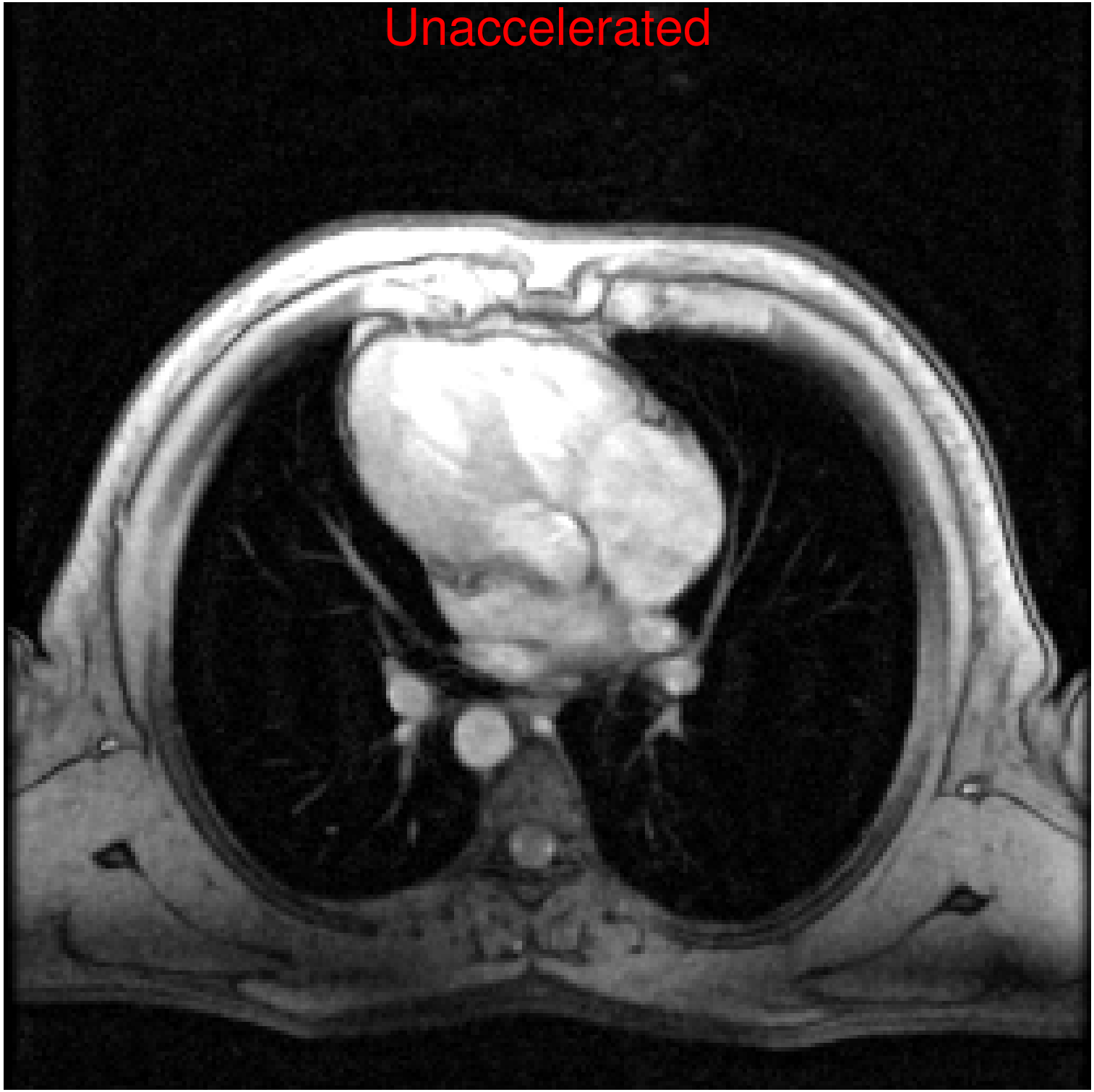


Accelerated X3,
Graph cuts reconstruction


Unaccelerated


Accelerated X3, SENSE reconstruction


Accelerated X3,Graph cuts reconstruction


Unaccelerated

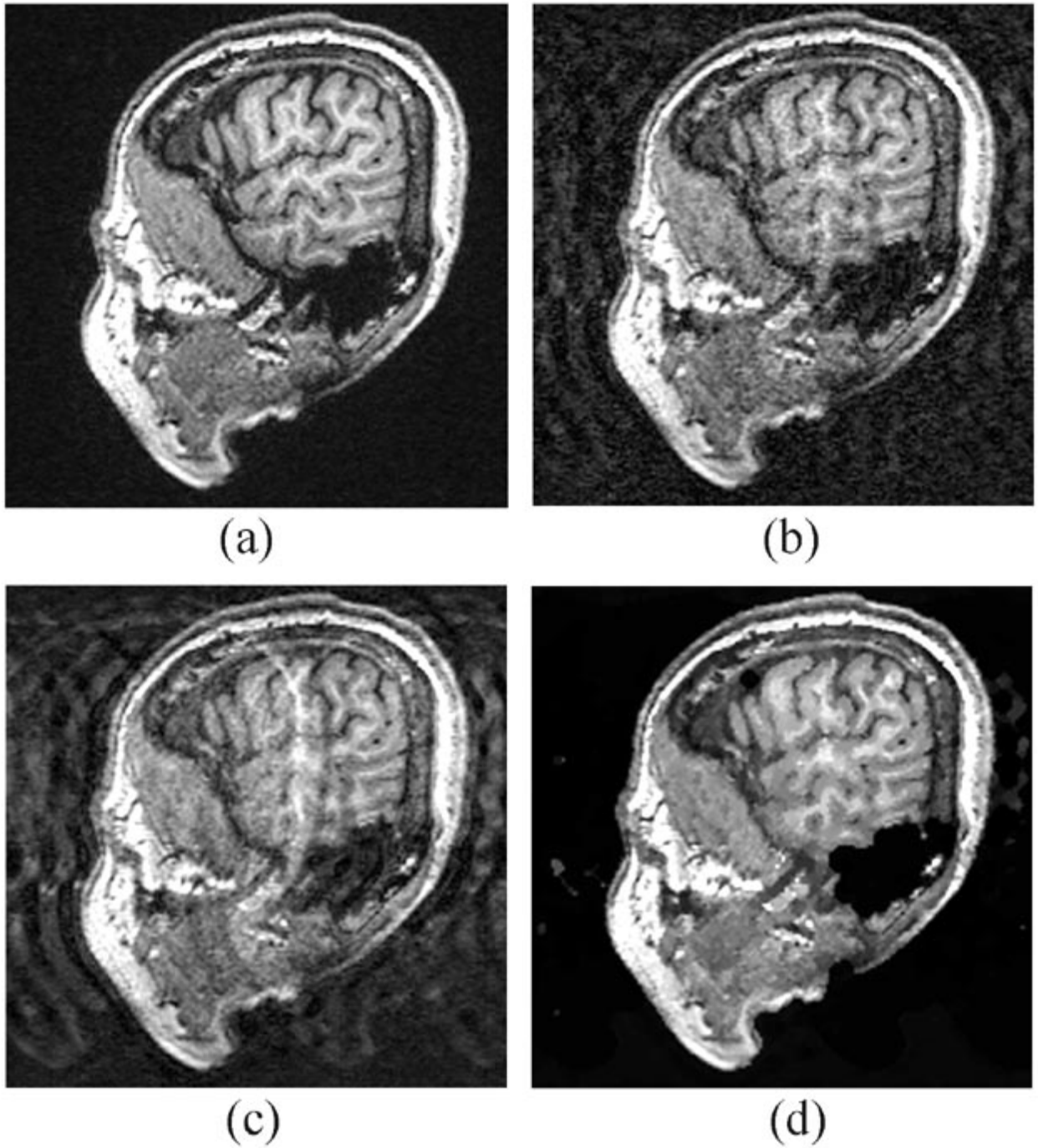
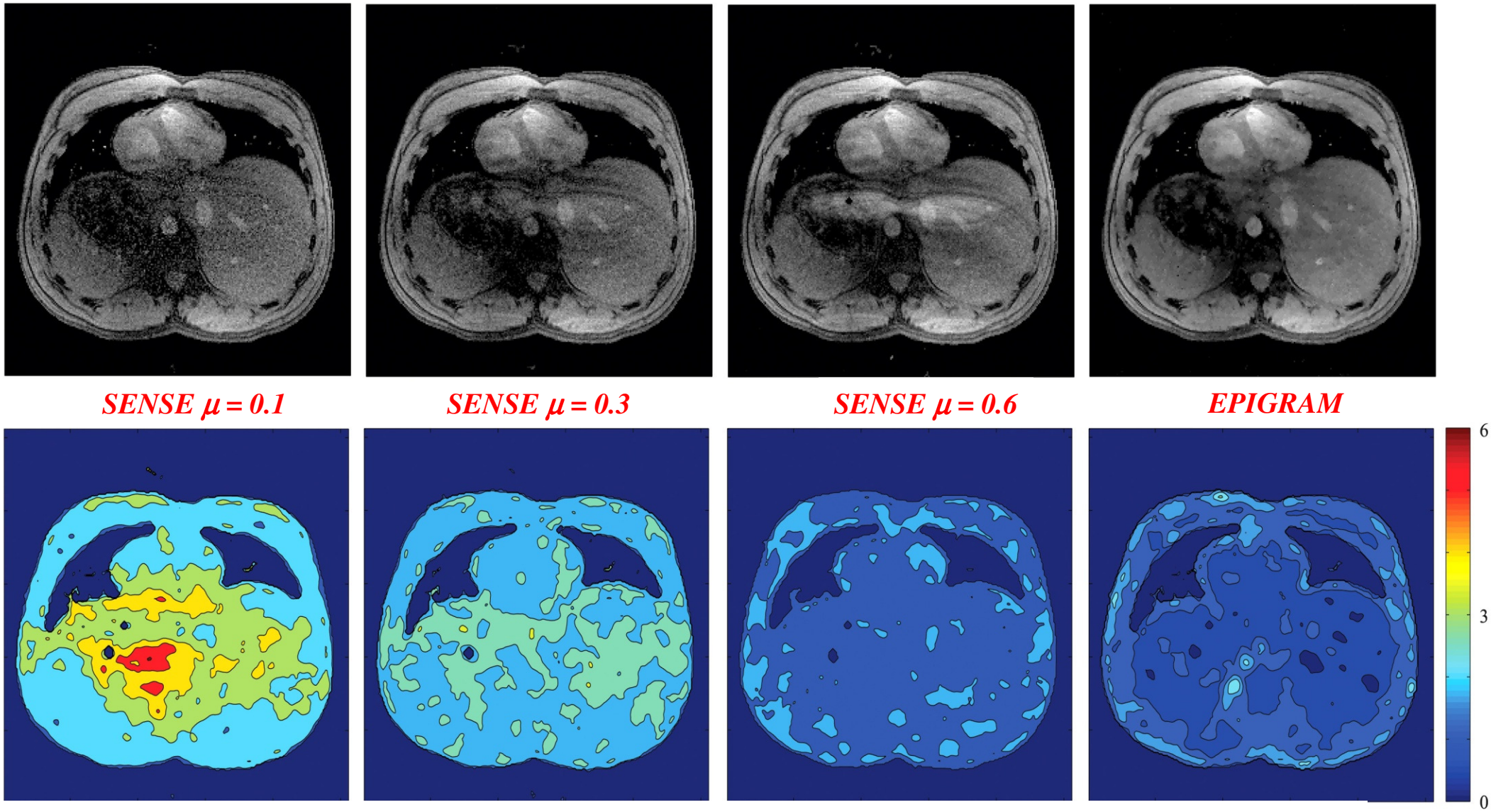


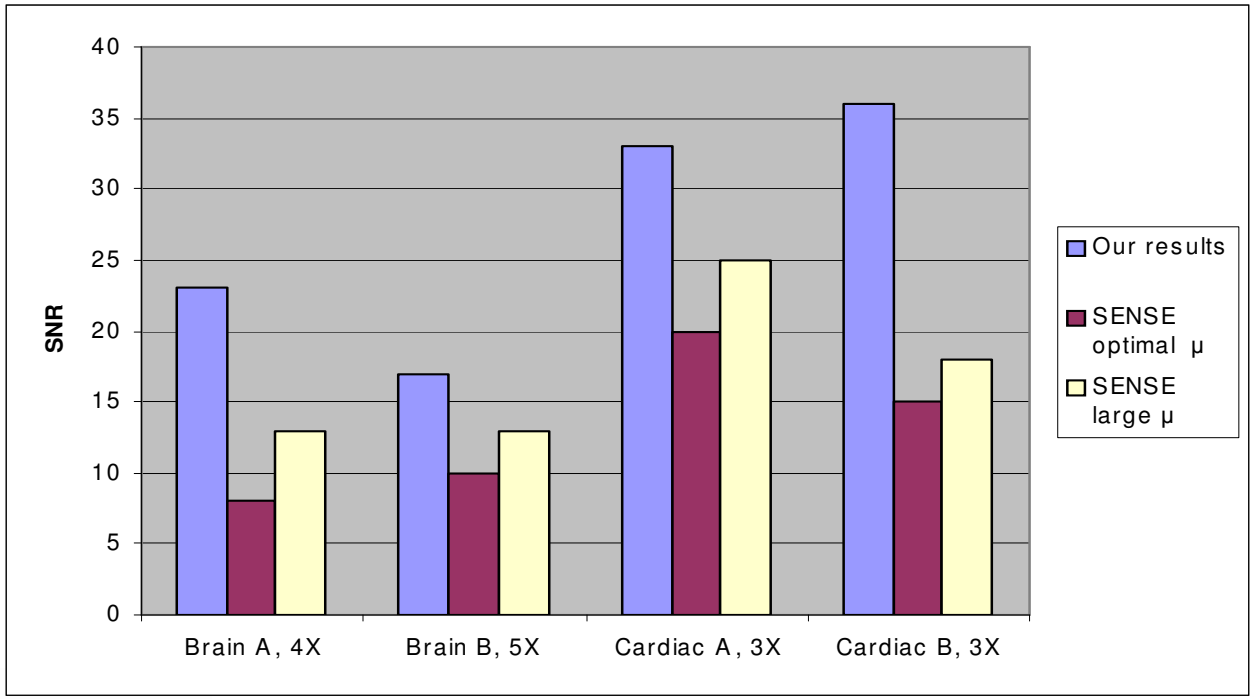
Figure 9: Brain B: In vivo brain result with R = 5, L=8. (a) Reference image (b) SENSE regularized with μ = 0.15 (c) SENSE regularized with μ = 0.3, (d) EPIGRAM reconstruction

Comparison with SENSE



In vivo results - SNR

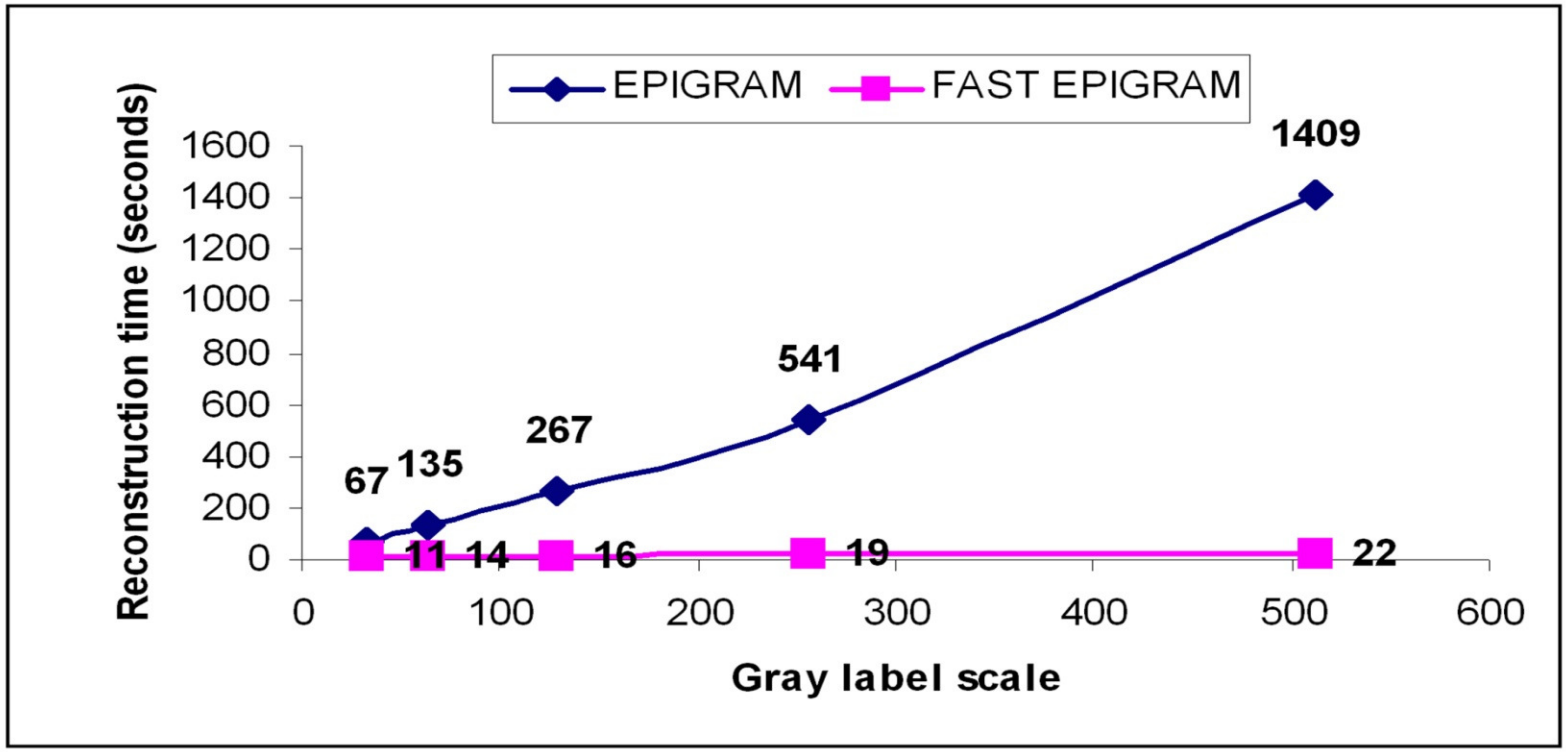
	R	Reg SENSE		EPIGRAM	
		mean SNR	Mean g	Mean SNR	Mean g
Brain A	4	8	4.6	23	1.7
Brain B	5	10	3.5	17	2.2
Cardiac A	3	20	2.3	33	1.5
Cardiac B	3	15	3.3	36	1.4



New Algorithm:

- Fast EPIGRAM – uses "jump moves" rather than "expansion moves"
  - Up to 50 times faster!

New, Faster Graph Cut Algorithm: Jump Moves



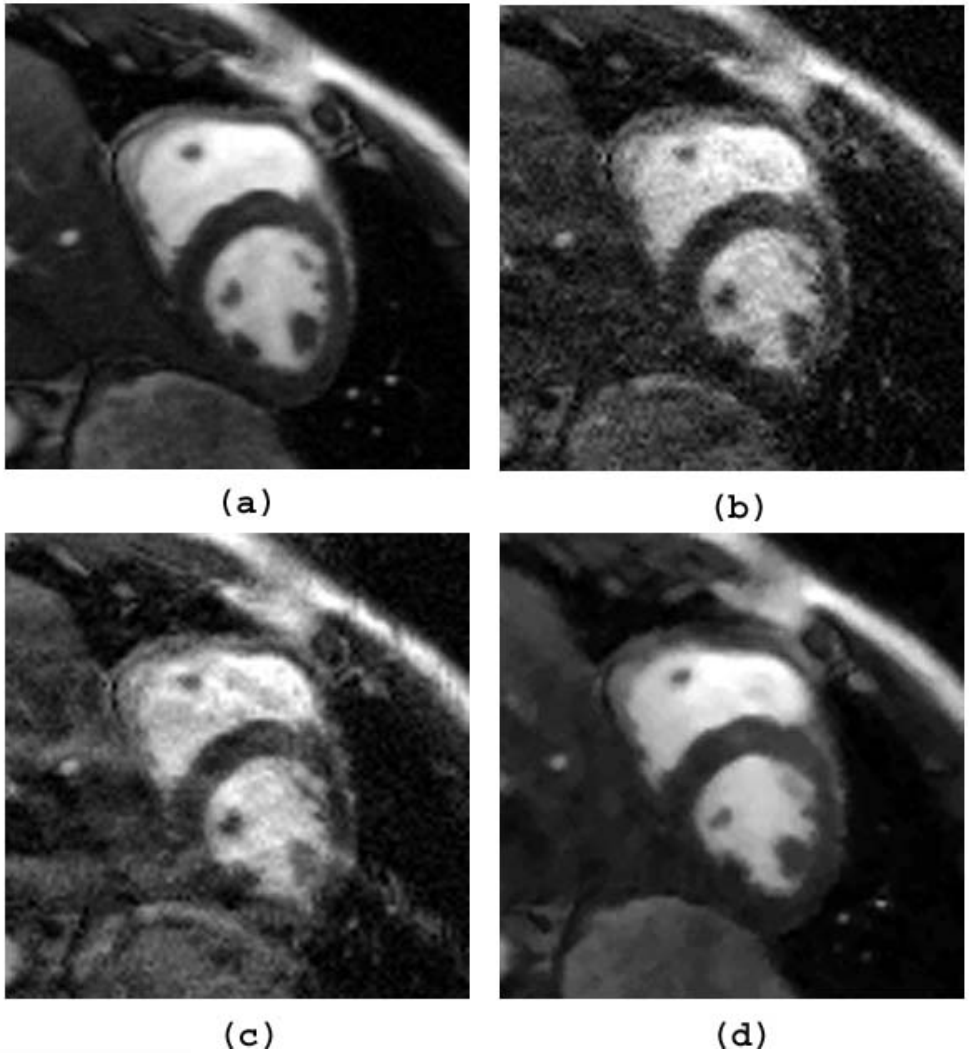
- *Reconstruction time of EPIGRAM (alpha expansion) vs Fast EPIGRAM (jump move)*
  - after 5 iterations over [32, 64, 128, 256, 512] gray scale labels
  image size 108x108 pixels.
- *Linear versus exponential growth in in reconstruction time*

Jump Move Results: Cardiac Imaging, R=4

•*reconstruction for cine SSFP at R = 4*

Reference:
Sum of squares

Regularized SENSE
(μ = 0.1)



Regularized SENSE
(μ = 0.5)

Fast EPIGRAM

References

Overview

- Ashish Raj. Improvements in MRI Using Information Redundancy. PhD thesis, Cornell University, May 2005.
- Website: http://www.cs.cornell.edu/~rdz/SENSE.htm

SENSE

- (1) Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: Sensitivity Encoding For Fast MRI. Magnetic Resonance in Medicine 1999; 42(5): 952-962.
- (2) Pruessmann KP, Weiger M, Boernert P, Boesiger P. Advances In Sensitivity Encoding With Arbitrary K-Space Trajectories. Magnetic Resonance in Medicine 2001; 46(4):638--651.
- (3) Weiger M, Pruessmann KP, Boesiger P. 2D SENSE For Faster 3D MRI. Magnetic Resonance Materials in Biology, Physics and Medicine 2002; 14(1):10-19.

References

ML-SENSE

• Raj A, Wang Y, Zabih R. A maximum likelihood approach to parallel imaging with coil sensitivity noise. IEEE Trans Med Imaging. 2007 Aug;26(8):1046-57

EPIGRAM

•Raj A, Singh G, Zabih R, Kressler B, Wang Y, Schuff N, Weiner M. Bayesian Parallel Imaging With Edge-Preserving Priors. Magn Reson Med. 2007 Jan;57(1):8-21

Regularized SENSE

•Lin F, Kwang K, BelliveauJ, Wald L. Parallel Imaging Reconstruction Using Automatic Regularization. Magnetic Resonance in Medicine 2004; 51(3): 559-67

References

Generalized Series Models

• Chandra S, Liang ZP, Webb A, Lee H, Morris HD, Lauterbur PC. Application Of Reduced-Encoding Imaging With Generalized-Series Reconstruction (RIGR) In Dynamic MR Imaging. J Magn Reson Imaging 1996; 6(5): 783-97.

• Hanson JM, Liang ZP, Magin RL, Duerk JL, Lauterbur PC. A Comparison Of RIGR And SVD Dynamic Imaging Methods. Magnetic Resonance in Medicine 1997; 38(1): 161-7.

Compressed Sensing in MR

• M Lustig, L Donoho, Sparse MRI: The application of compressed sensing for rapid mr imaging. Magnetic Resonance in Medicine. v58 i6

Weill Cornell Medical College

IMAGING DATA EVALUATION AND ANALYTICS LAB (IDEAL)

CS5540: Computational Techniques for Analyzing Clinical Data
Lecture 16:

Accelerated MRI Image Reconstruction

Ashish Raj, PhD

Image Data Evaluation and Analytics Laboratory (IDEAL)

Department of Radiology

Weill Cornell Medical College

New York