Can this be generalized?

- NP-hard for Potts model [K/BVZ 01]
- Two main approaches
  1. Exact solution [Ishikawa 03]
    - Large graph, convex $V$ (arbitrary $D$)
    - Not the considered the right prior for vision
  2. Approximate solutions [BVZ 01]
    - Solve a binary labeling problem, repeatedly
    - Expansion move algorithm
Exact construction for L1 distance

- Graph for 2 pixels, 7 labels:
  - 6 non-terminal vertices per pixel \((6 = 7 - 1)\)
  - Certain edges (vertical green in the figure) correspond to different labels for a pixel
    - If we cut these edges, the right number of horizontal edges will also be cut

- Can be generalized for convex \(V\) (arbitrary \(D\))
Convex over-smoothing

- Convex priors are widely viewed in vision as inappropriate ("non-robust")
  - These priors prefer globally smooth images
    • Which is almost never suitable
- This is not just a theoretical argument
  - It’s observed in practice, even at global min
Appropriate prior?

- We need to avoid over-penalizing large jumps in the solution
- This is related to outliers, and the whole area of robust statistics
- We tend to get structured outliers in images, which are particularly challenging!
Getting the boundaries right

Right answers

Graph cuts
Expansion move algorithm

- Make green expansion move that most decreases $E$
  - Then make the best blue expansion move, etc
  - Done when no $\alpha$-expansion move decreases the energy, for any label $\alpha$
  - See [BVZ 01] for details
Local improvement vs. Graph cuts

- Continuous vs. discrete
  - No floating point with graph cuts

- Local min in line search vs. global min

- Minimize over a line vs. hypersurface
  - Containing $O(2^n)$ candidates

- Local minimum: weak vs. strong
  - Within 1% of global min on benchmarks!
  - Theoretical guarantees concerning distance from global minimum
    - 2-approximation for a common choice of $E$
2-approximation for Potts model

optimal solution $f^*$

local minimum $\hat{f}$

Summing up over all labels:

$$E(\hat{f}) \leq E(f^*) + E_\partial(f^*) \leq 2E(f^*)$$
Binary sub-problem

Input labeling
Expansion move
Binary image
Expansion move energy

Goal: find the binary image with lowest energy

Binary image energy $E(b)$ is restricted version of original $E$
Depends on $f, \alpha$
Regularity

- The binary energy function

\[ \sum_{p} B_p(x_p) + \sum_{p,q} B_{p,q}(x_p, x_q) \]

is regular [KZ 04] if

\[ B_{p,q}(0, 0) + B_{p,q}(1, 1) \leq B_{p,q}(0, 1) + B_{p,q}(1, 0) \]

- This is a special case of submodularity