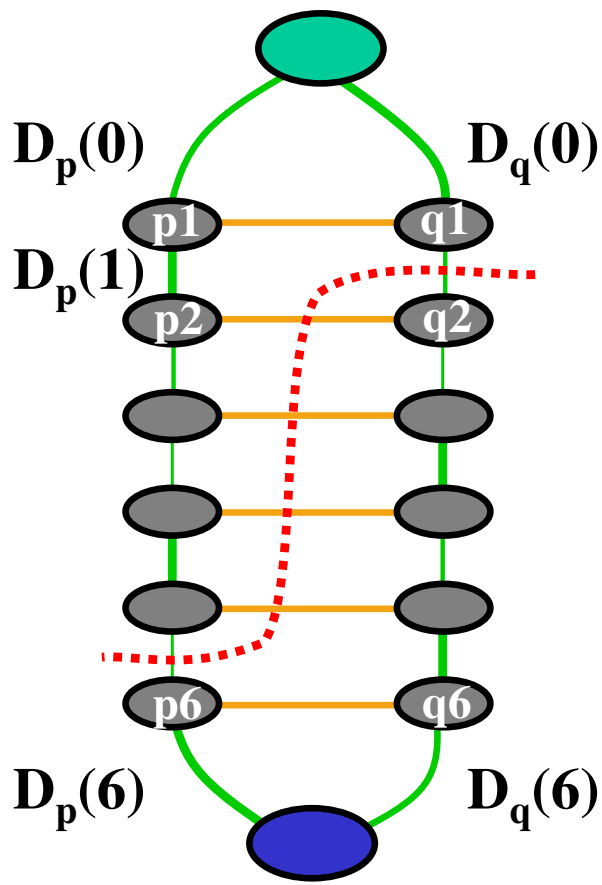
Can this be generalized?

- NP-hard for Potts model [K/BVZ 01]
- Two main approaches

1. Exact solution [Ishikawa 03]
   - Large graph, convex V (arbitrary D)
   - Not the considered the right prior for vision
2. Approximate solutions [BVZ 01]
   - Solve a binary labeling problem, repeatedly
   - Expansion move algorithm

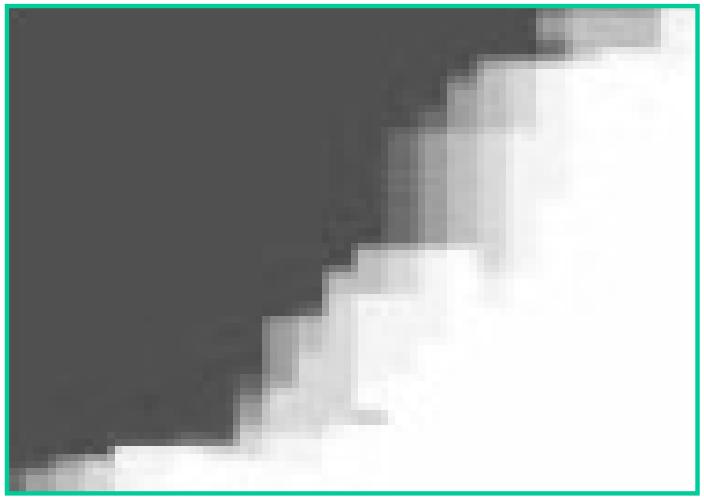
Exact construction for L1 distance

- Graph for 2 pixels, 7 labels:
  - 6 non-terminal vertices per pixel (6 = 7 – 1)
  - Certain edges (vertical green in the figure) correspond to different labels for a pixel
    - If we cut these edges, the right number of horizontal edges will also be cut
- Can be generalized for convex V (arbitrary D)



Convex over-smoothing

- Convex priors are widely viewed in vision as inappropriate ("non-robust")
  - These priors prefer globally smooth images
    - Which is almost never suitable
- This is not just a theoretical argument
  - It's observed in practice, even at global min



Appropriate prior?

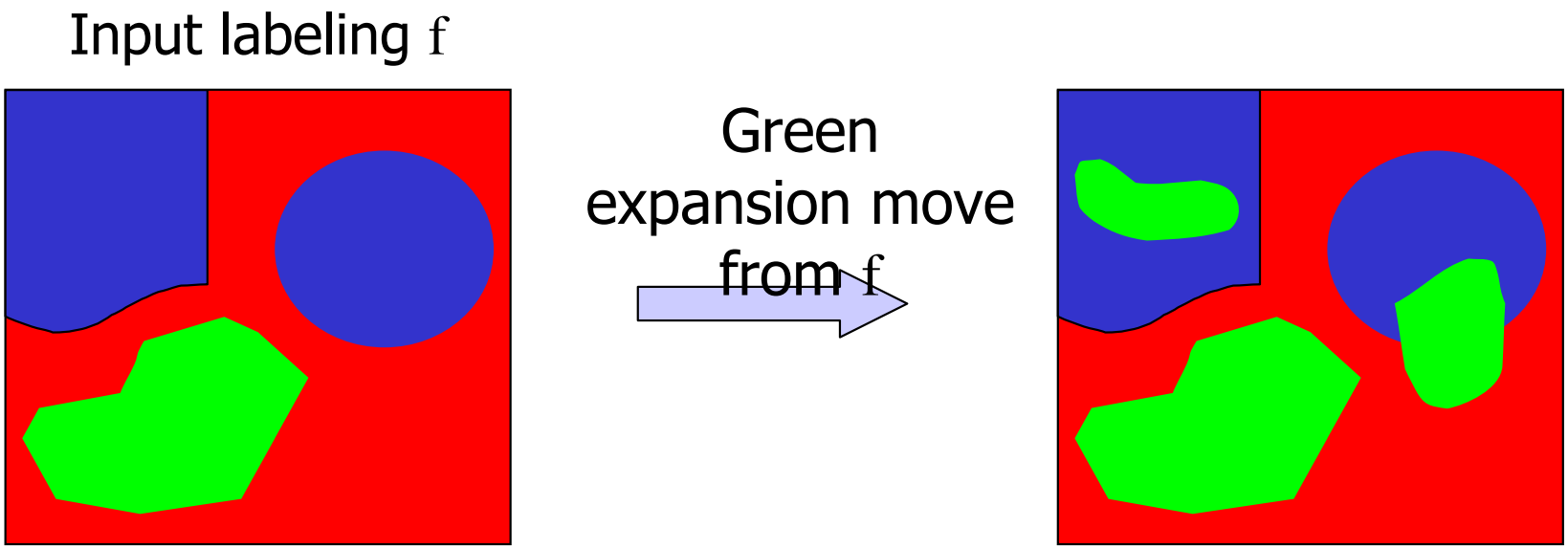
- We need to avoid over-penalizing large jumps in the solution
- This is related to outliers, and the whole area of robust statistics
- We tend to get structured outliers in images, which are particularly challenging!

Getting the boundaries right

Right answers

Graph cuts

Expansion move algorithm

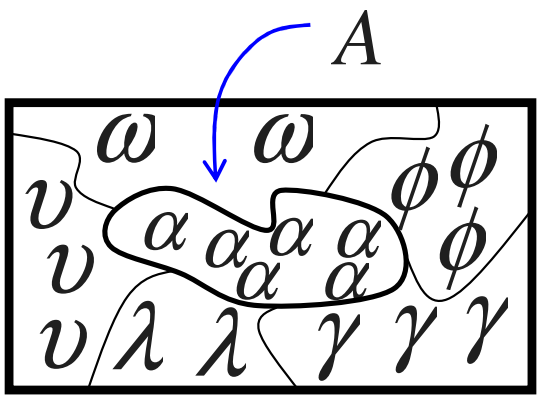


- Make green expansion move that most decreases E
  - Then make the best blue expansion move, etc
  - Done when no α-expansion move decreases the energy, for any label α
  - See [BVZ 01] for details

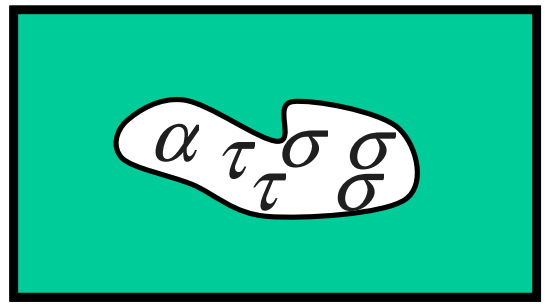
Local improvement vs. Graph cuts

- Continuous vs. discrete
  - No floating point with graph cuts
- Local min in line search vs. global min
- Minimize over a line vs. hypersurface
  - Containing $O(2^n)$ candidates
- Local minimum: weak vs. strong
  - Within 1% of global min on benchmarks!
  - Theoretical guarantees concerning distance from global minimum
    - 2-approximation for a common choice of E

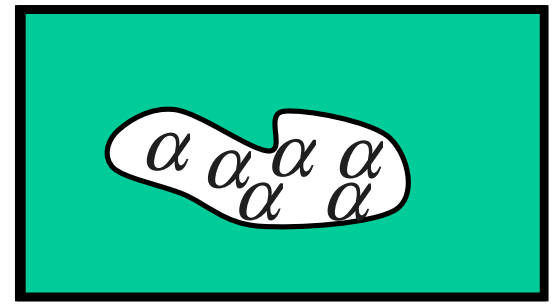
2-approximation for Potts model



optimal solution f^*



local minimum $\hat{f}$



$$f^{\alpha} = \begin{cases} \alpha & p \in A \\ \hat{f}_p & p \notin A \end{cases}$$

$$\hat{f} \;\alpha\; f^{\alpha} \quad \Longrightarrow \quad E\left(\hat{f}\right) \le E\left(f^{\alpha}\right)$$

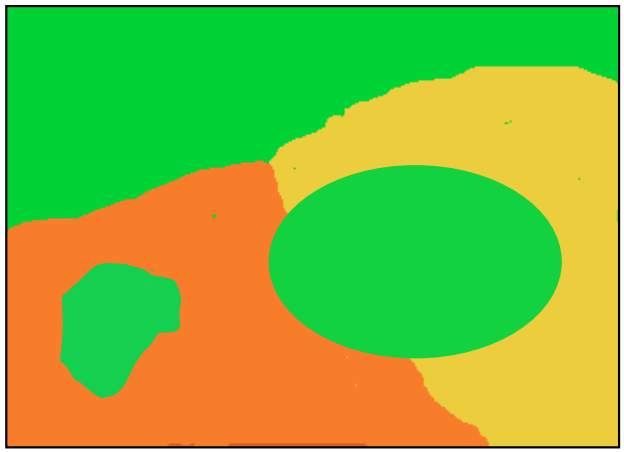
$$E_A\left(\hat{f}\right) \le E_A\left(f^{\alpha}\right) \le E_A\left(f^*\right)$$

Summing up over all labels:

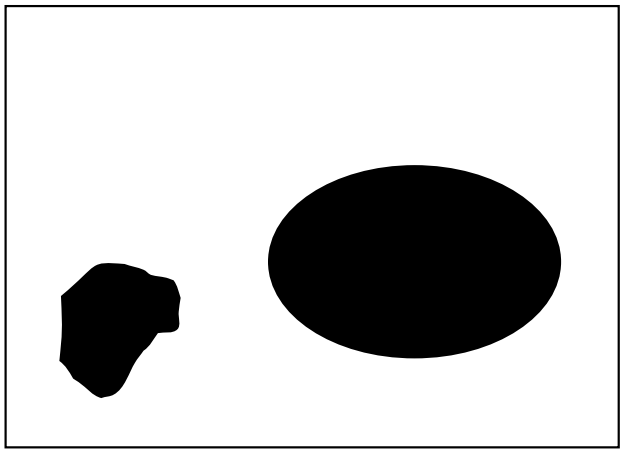
$$E\left(\hat{f}\right) \le E\left(f^*\right) + E_{\partial}\left(f^*\right) \le 2E\left(f^*\right)$$

Binary sub-problem

Input labeling

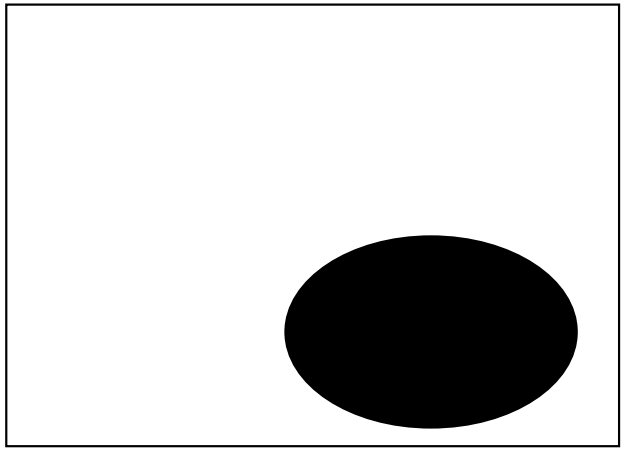


Expansion move



Binary image

Expansion move energy



Goal: find the binary image with lowest energy

Binary image energy **E**(**b**) is restricted version of original **E**

Depends on $\mathbf{f}, \alpha$

Regularity

- The binary energy function

$$\sum_p B_p(x_p) + \sum_{p,q} B_{p,q}(x_p, x_q)$$

is *regular* [KZ 04] if

$$B_{p,q}(0,0) + B_{p,q}(1,1) \leq B_{p,q}(0,1) + B_{p,q}(1,0)$$

- This is a special case of submodularity