Optimal solution of binary problems

Much material taken from:

- **Olga Veksler, University of Western Ontario**

http://www.csd.uwo.ca/faculty/olga/

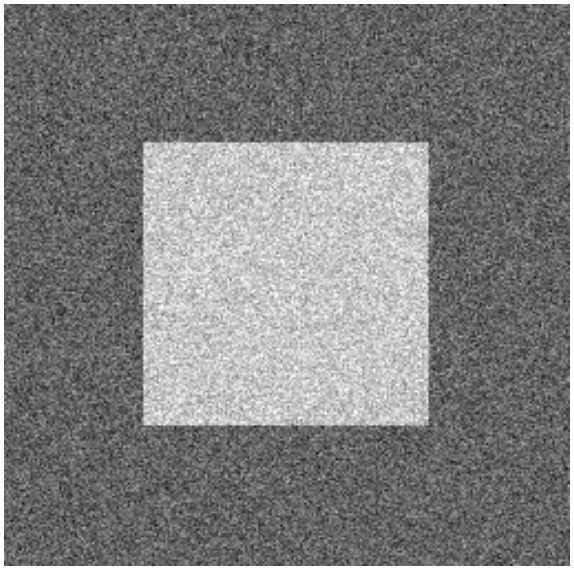
Announcements

- Project proposal due March 31
  - Email to RDZ
  - One or two paragraphs
- Project will be due the last week of classes, or perhaps a bit later
- Ashish will lecture on Friday

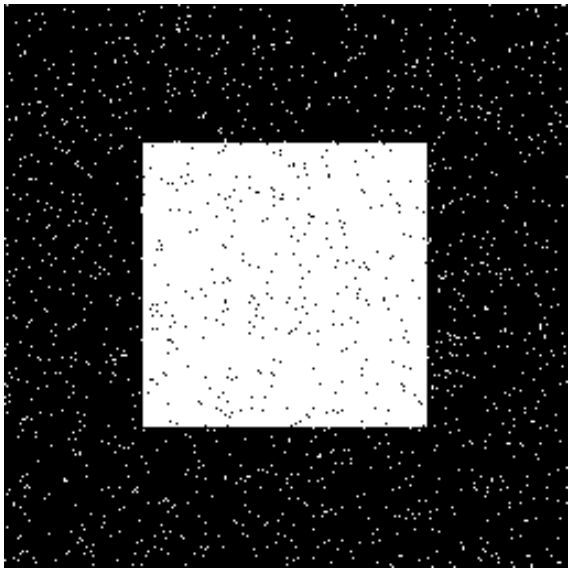
Motivating example

- Suppose we want to find a bright object against a dark background
  - But some of the pixel values are slightly wrong

Input



Best thresholded image



Optimization viewpoint

- Find best (least expensive) binary image
  - Costs: C1 (labeling) and C2 (boundary)
- C1: Labeling a dark pixel as foreground
  - Or, a bright pixel as background
- If we only had labeling costs, the cheapest solution is the thresholded output
- C2: The length of the boundary between foreground and background
  - Penalizes isolated pixels or ragged boundaries

MAP-MRF energy function

- Generalization of C2 is $\sum_{p,q} V_{p,q}(x_p, x_q)$
  - Think of V as the cost for two adjacent pixels to have these particular labels
  - For binary images, the natural cost is uniform
- Bayesian energy function:

$$E(x_1, \ldots, x_n) = \sum_p D_p(x_p) + \sum_{p,q} V_{p,q}(x_p, x_q)$$

Generalizations

- Many vision problems have this form
- The optimization techniques are specific to these energy function, but not to images
  - See: [Kleinberg & Tardos JACM 02]
- Historically solved by gradient descent or related methods (e.g. annealing)
  - Optimization method and energy function are not independent choices!
  - Use the most specific method you can
    - And, be prepared to tweak your problem

Binary labeling problems

- Consider the case of two labels only
  - Surprisingly important
    - Lots of nice applications
    - Same basic ideas used for more labels
- There is a fast exact solution!
  - Turn a problem you don't know how to solve into a problem you do (reduction)
    - Due to Hammer (1965) originally
    - Job scheduling problems: Stone (1977)
    - Images: Greig, Porteus & Seheult (1989)