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# CS 5430

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## Hyperproperties

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# Security Policies Today

## Confidentiality

“Protection of assets from unauthorized disclosure”

## Integrity

“Protection of assets from unauthorized modification”

## Availability

“Protection of assets from loss of use”

Formalize and verify any security policy? 

# Program Correctness ca. 1970s

- Partial correctness (If program terminates, it produces correct output)
- Termination
- Total correctness (Program terminates and produces correct output)
- Mutual exclusion
- Deadlock freedom
- Starvation freedom

???

# Safety and Liveness Properties

Intuition [Lamport 1977]:

## **Safety:**

“Nothing bad happens”

- Partial correctness  
Bad thing: program terminates with incorrect output
- Access control  
Bad thing: subject completes operation without required rights

## **Liveness:**

“Something good happens”

- Termination  
Good thing: termination
- Guaranteed service  
Good thing: service rendered

# Properties

**Trace:** Sequence of execution states

$$t = s_0s_1\ldots$$

**Property:** Set of infinite traces

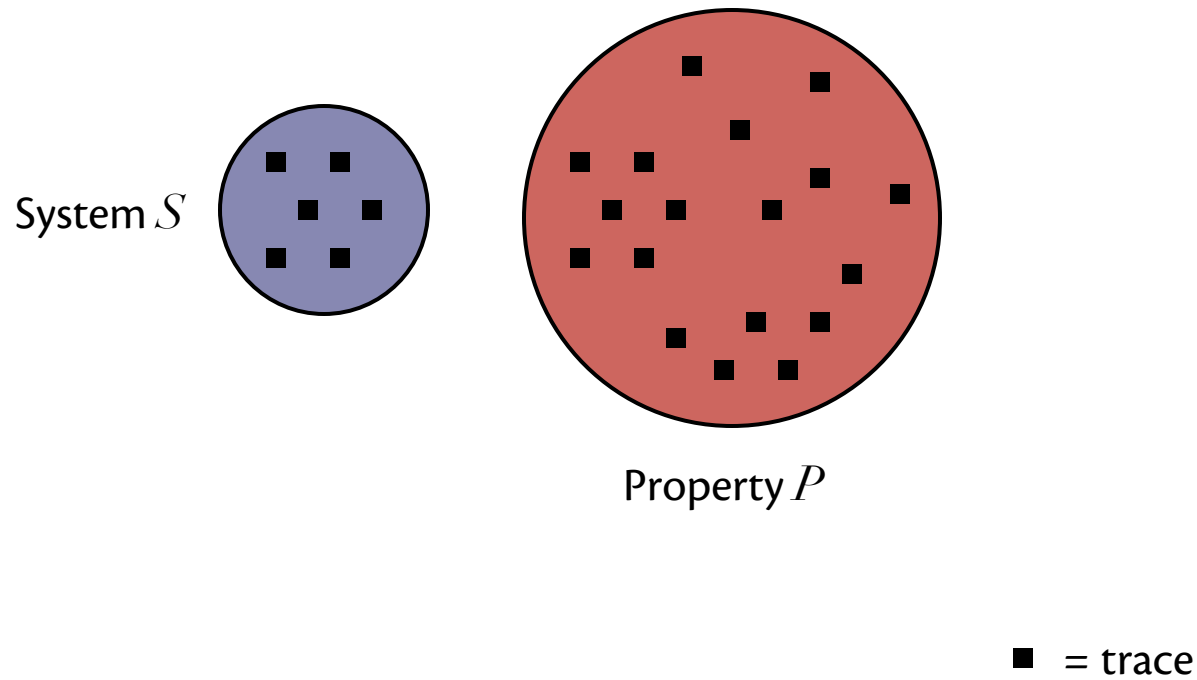
Trace  $t$  satisfies property  $P$  iff  $t$  is an element of  $P$

➔ Satisfaction depends on the trace alone

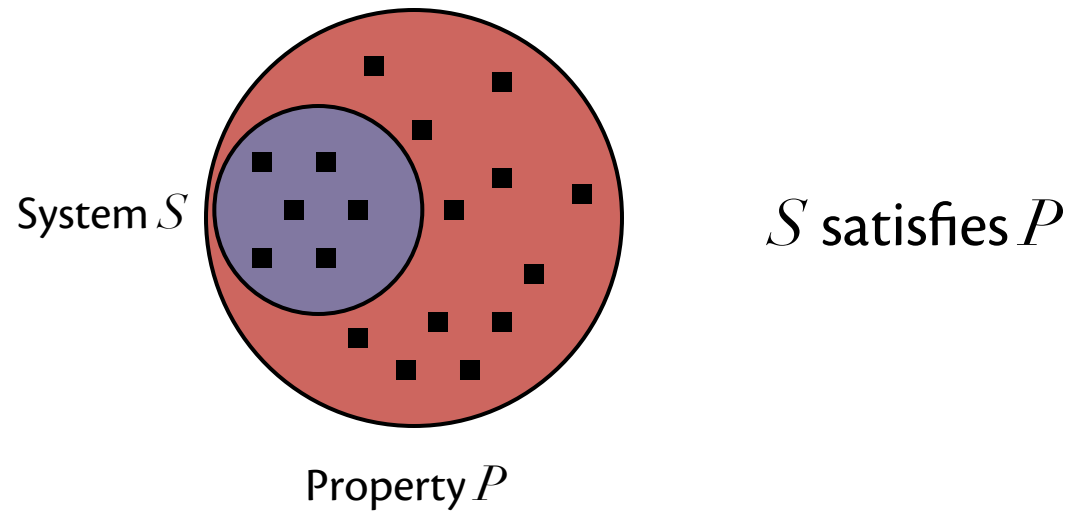
**System:** Also a set of traces

System  $S$  satisfies property  $P$  iff all traces of  $S$  satisfy  $P$

# Properties

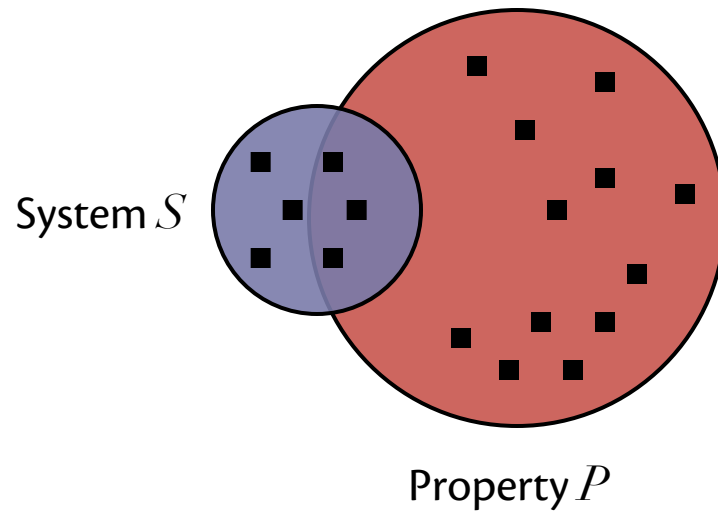


# Properties



■ = trace

# Properties



$S$  does not satisfy  $P$

■ = trace



# Safety and Liveness Properties

Formalized:

**Safety property** [Lamport 1985]

Bad thing = trace prefix

**Liveness property** [Alpern and Schneider 1985]

Good thing = trace suffix

# Success!

Alpern and Schneider (1985, 1987):

**Theorem.** *Every property is the intersection of a safety property and a liveness property.*

**Theorem.** *Safety proved by invariance.*

**Theorem.** *Liveness proved by well-foundedness.*

**Theorem.** *Topological characterization:*

*Safety = closed sets*

*Liveness = dense sets*

Formalize and verify any property? 

# Back to Security Policies

Formalize and verify any property?



Formalize and verify any security policy?



Security policy  $\stackrel{?}{=}$  Property

# Information Flow is not a Property

## Secure information flow:

Secret inputs are not leaked to public outputs

`p := 1;`



`p := s;`



`if (s) then p := 1 else p := 0;`



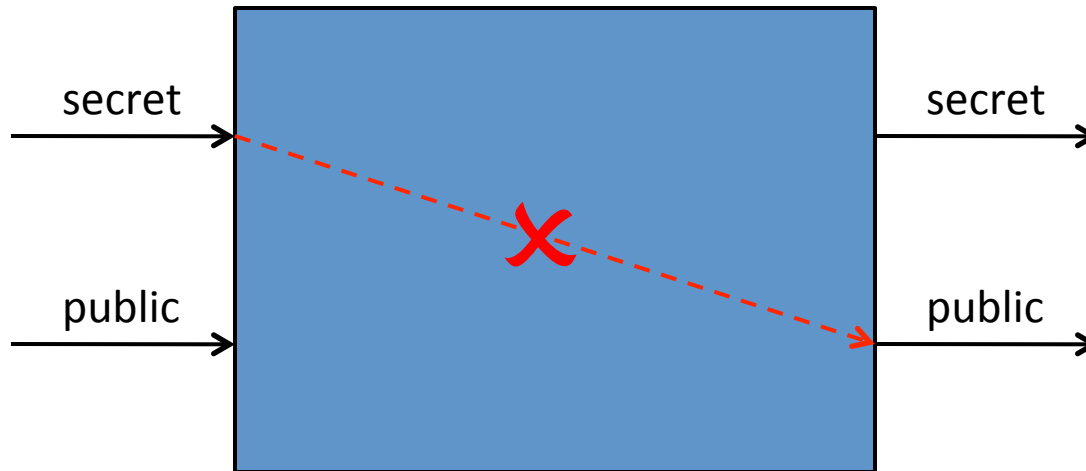
`if (s) then {consume power} else {don't};`



# Information Flow is not a Property

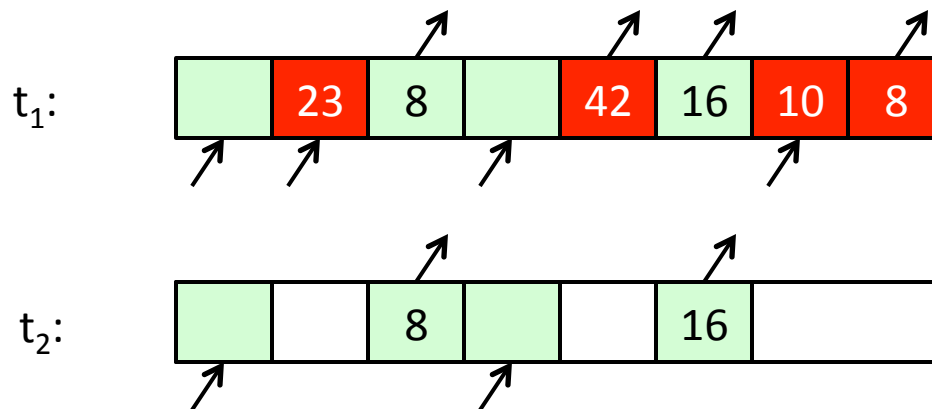
## Secure information flow:

Secret inputs are not leaked to public outputs



# Information Flow is not a Property

**Noninterference** [Goguen and Meseguer 1982]: Commands of high security users have no effect on observations of low security users



*Not safety!*

Satisfaction depends on **pairs** of traces ...so not a property

# Service Level Agreements are not Properties

**Service level agreement:** Acceptable performance of system

*Not liveness!*

**Average response time:** Average time, over all executions, to respond to request has given bound

- Satisfaction depends on **all** traces of system ...not a property

Any security policy that stipulates relations among traces is not a property

➔ Need satisfaction to depend on *sets* of traces [McLean 1996]

# Hyperproperties

A **hyperproperty** is a set of properties

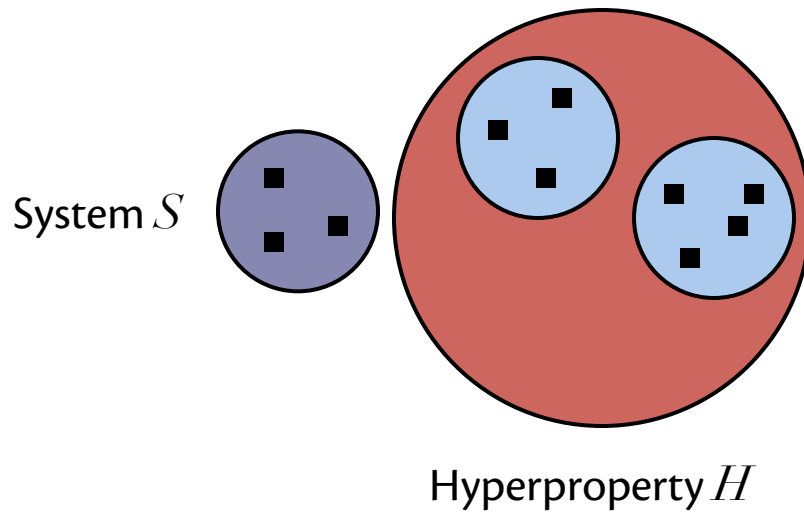
[Clarkson and Schneider 2008, 2010]

A system  $S$  **satisfies** a hyperproperty  $H$   
iff  $S$  is an element of  $H$

...a hyperproperty specifies exactly the allowed sets of traces



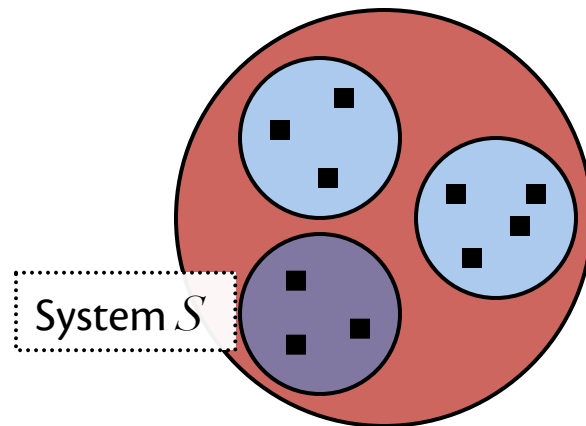
# Hyperproperties



$S$  does not satisfy  $H$

■ = trace

# Hyperproperties



Hyperproperty  $H$

$S$  satisfies  $H$

■ = trace

# Hyperproperties

Security policies are hyperproperties!

- **Information flow:** Noninterference, relational noninterference, generalized noninterference, observational determinism, self-bisimilarity, probabilistic noninterference, quantitative leakage
- **Service-level agreements:** Average response time, time service factor, percentage uptime
- ...

# Beyond Hyperproperties?

- *Security policies* are predicates on *systems*
- Hyperproperties are the *extensions* of those predicates

➔ Hyperproperties are expressively complete  
(for predicates, systems, and trace semantics)

# Other System Models

- Relational semantics
- Labeled transition systems
- State machines
- Probabilistic systems

...can define hyperproperties for all these

# Probabilistic Hyperproperties

To incorporate probability:

- Assume probability on state transitions
- Construct probability measure on traces [Halpern 2003]
- Use measure to express hyperproperties

We've expressed:

- **Probabilistic noninterference** [Gray and Syverson 1998]
- Quantitative leakage
- Channel capacity

# Hyperproperties

- Safety and liveness?
- Verification?

# Safety

Safety proscribes “bad things”

- A bad thing is **finitely observable** and **irremediable**
- $S$  is a safety property [Lamport 85] iff

$$(\forall t \notin S : (\exists b \leq t : (\forall u \geq b : u \notin S)))$$

$b$  is a finite trace





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$b$  is a finite trace

- $S$  is a **safety hyperproperty** (“hypersafety”) iff

$$(\forall T \notin S : (\exists B \leq T : (\forall U \geq B : U \notin S)))$$

$B$  is a finite set of finite traces

# Prefix Ordering

An **observation** is a finite set of finite traces

Intuition: Observer sees a set of partial executions

$M \leq T$  ( $M$  is a **prefix** of  $T$ ) iff:

- $M$  is an observation, and
- $\forall m \in M : (\exists t \in T : m \leq t)$
- If observer watched longer,  $M$  could become  $T$

# Safety Hyperproperties

## **Noninterference** [Goguen and Meseguer 1982]

Bad thing is a pair of traces where removing high commands does change low observations

## **Observational determinism** [Roscoe 1995, Zdancewic and Myers 2003]

Bad thing is a pair of traces that cause system to look nondeterministic to low observer

...

# Liveness

Liveness prescribes “good things”

- A good thing is **always possible** and **possibly infinite**
- $L$  is a liveness property [AS85] iff
$$(\forall t : (\exists g \geq t : g \in L))$$

$t$  is a finite trace



# Liveness

Liveness prescribes “good things”

- A good thing is **always possible** and **possibly infinite**
- $L$  is a liveness property [AS85] iff

$$(\forall t : (\exists g \geq t : g \in L))$$



$t$  is a finite trace

- $L$  is a **liveness hyperproperty** (“hyperliveness”) iff

$$(\forall T : (\exists G \geq T : G \in \mathbf{L}))$$



$T$  is a finite set of finite traces

# Liveness Hyperproperties

## Average response time

Good thing is that average time is low enough

## Possibilistic information flow

Class of policies requiring “alternate possible explanations” to exist

e.g. **noninference**

**Theorem.** *All PIF policies are hyperliveness.*

# Relating Properties and Hyperproperties

Can **lift** property  $T$  to hyperproperty  $[T]$

Satisfaction is equivalent iff  $[T] = \text{powerset}(T)$

**Theorem.**  *$S$  is safety implies  $[S]$  is hypersafety.*

**Theorem.**  *$L$  is liveness implies  $[L]$  is hyperliveness.*

...Verification techniques for safety and liveness carry forward to hyperproperties



# Safety and Liveness is a Basis (still)

**Theorem.** *Every hyperproperty is the intersection of a safety hyperproperty and a liveness hyperproperty.*

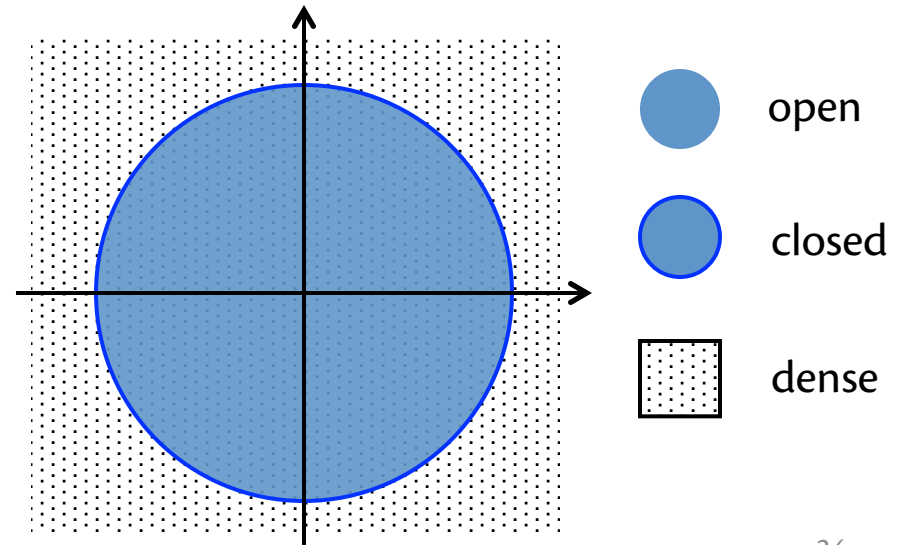
A fundamental basis...

# Topology

**Open set:** Can always “wiggle” from point and stay in set

**Closed set:** “Wiggle” might move outside set

**Dense set:** Can always “wiggle” to get into set



# Topology of Hyperproperties

For **Plotkin topology** on properties [AS85]:


- Safety = closed sets
- Liveness = dense sets

**Theorem.** *Hypersafety = closed sets.*

**Theorem.** *Hyperliveness = dense sets.*

**Theorem.** *Our topology on hyperproperties is equivalent to the lower Vietoris construction applied to the Plotkin topology.*

# Stepping Back...

- Safety and liveness? 
- Verification?

# Logic and Verification

Temporal logic: LTL, CTL\*?

- Highly successful for trace properties
- But not for security policies [McLean 1994, Alur et al. 2006]
- Let's hyper-ize... with quantification over multiple traces

# Syntax

**LTL:** [Pnueli 1977]

$$\phi ::= p \mid \neg \phi \mid \phi_1 \vee \phi_2 \mid \dots \mid X \phi \mid \phi_1 U \phi_2 \mid \dots \mid G \phi \mid \dots$$

State propositions:  $x\text{-equals-}42$

**HyperLTL:** [Koleini, Clarkson, Micinski 2013]

$$\psi ::= \text{At: } \psi \mid \text{Et: } \psi \mid \phi$$

State propositions annotated with trace variable:  $x\text{-equals-}42_t$

...LTL is a fragment of HyperLTL

# Examples

**Observational determinism** [Zdancewic and Myers 2003]:

$$\text{At: Au: } t[0] =_L u[0] \Rightarrow t =_L u$$

$t[0] =_L u[0]$  is sugar for  $\bigwedge_{p \in L} p_t \Leftrightarrow p_u$   
(first state in both traces agrees on all propositions in L)

$t =_L u$  is sugar for  $G (t[0] =_L u[0])$   
(both traces agrees on all propositions in L)

Note: multiple paths in scope; syntax that reads like the “normal” math written in noninterference papers.

# Examples

**Noninference** [McLean 1994]:

$$At: Eu: t=_L u \wedge G \text{ no-high}_u$$

state-based variant of GM noninterference

Can also express noninterference itself.

And GNI, restrictiveness, separability, forward  
correctability...



# Semantics

## LTL:

- formula modeled by single trace:  $t \models \phi$
- system modeled by set  $T$  of traces

## HyperLTL:

- formula modeled by **set** of traces (*actually, set of named traces i.e. valuation or environment*)
- system still modeled by set  $T$  of traces, which is what quantifiers range over:

$$\Pi \models A t : \psi \text{ iff for all } \tau \text{ in } T, \text{ have } \Pi, t=\tau \models \psi$$

# Semantics

$\Pi \models A t : \psi$  iff for all  $\tau$  in  $T$ , have  $\Pi, t=\tau \models \psi$

$\Pi \models E t : \psi$  iff exists  $\tau$  in  $T$ , s.t.  $\Pi, t=\tau \models \psi$

$\Pi \models p_t$  iff  $p \in \Pi(t)[0]$

$\Pi \models \neg \phi$  iff  $\Pi \models \phi$  doesn't hold

$\Pi \models \phi_1 \vee \phi_2$  iff  $\Pi \models \phi_1$  or  $\Pi \models \phi_2$

$\Pi \models X \phi$  iff  $\Pi[1..] \models \phi$

$\Pi \models \phi_1 \cup \phi_2$  iff there exists  $i \geq 0$  s.t.  $\Pi[i..] \models \phi_2$  and  
for all  $j$  where  $0 \leq j < i$ , have  $\Pi[j..] \models \phi_1$

# Model Checking

- Adapts LTL algorithm based on Büchi automata  
[Wolper et al. 1983, Lichtenstein and Pnueli 1985, Vardi and Wolper 1994, ...]
- Prototype...
  - builds automata using self-composition [Barthe et al. 2004],
  - then outsources to GOAL [Tsay et al. 2007] for automata constructions
- Supports fragment of HyperLTL
  - Up to one quantifier alternation, e.g. AE, AAE, EA
  - Suffices for all our information-flow examples
- Yields verification methodology for any *linear-time* hyperproperty

# Model Checking: Complexity

- Fragment with 1 alternation:
  - Exponential in size of system and
  - Doubly exponential in size of formula
- Full HyperLTL:
  - PSPACE-hard
  - Reduction from quantified propositional temporal logic (QPTL)

...price of security? Or do we need to be more clever?

# Other Hyper Temporal Logics

- **HyperCTL\*** [Finkbeiner et al. 2013]
  - Like HyperLTL, but quantifiers can be nested
  - Model checking is  
NSPACE( $f(\text{size of system})$ )-complete  
where  $f$  involves a tower of exponentials... ☹
- **“Hyper modal  $\mu$ -calculus”**
  - Polyadic modal  $\mu$ -calculus [Andersen 1994]
  - Used by Milushev and Clarke [2012] for *incremental hyperproperties*

# Stepping Back...

- Safety and liveness? ✓
- Verification?
  - Model-checking (expensive) ✓
  - Reduce to trace properties
  - Refinement

# Verification of 2-Safety

**2-safety:** “Property that can be refuted by observing two finite traces” [Terauchi and Aiken 2005]

Methodology:

- Transform system with **self-composition construction** [Barthe, D’Argenio, and Rezk 2004]
- Verify safety property of transformed system
  - Implies 2-safety property of original system

...Reduction from hyperproperty to property

# *k*-Safety Hyperproperties

A ***k*-safety hyperproperty** is a safety hyperproperty in which the bad thing never has more than *k* traces

$$(\forall T \notin S : (\exists B \leq T : |B| \leq k \wedge (\forall U \geq B : B \notin S)))$$

Examples:

- **1-hypersafety**: the lifted safety properties
- **2-hypersafety**: Terauchi and Aiken's 2-safety properties
- ***k*-hypersafety**:  $SEC(k)$  = "System can't, across all runs, output all shares of a *k*-secret sharing"
- **Not *k*-hypersafety for any *k***:  $SEC = \bigcup_k SEC(k)$



# Verifying $k$ -Hypersafety

**Theorem.** *Any  $k$ -safety hyperproperty of  $S$  is equivalent to a safety property of  $S^k$ .*

→ Yields methodology for  $k$ -hypersafety

- Incomplete for hypersafety
- Hyperliveness? In general?

# Refinement Revisited

## Stepwise refinement:

- Development methodology for properties
  - Start with specification and high-level (abstract) program
  - Repeatedly **refine** program to lower-level (concrete) program
- Techniques for refinement well-developed

Long-known those techniques don't work for security policies—i.e., hyperproperties

- Develop new techniques?
- Reuse known techniques?

# Refinement Revisited

**Theorem.** *Known techniques work with all hyperproperties that are subset-closed.*

**Theorem.** *All safety hyperproperties are subset-closed.*

➔ Stepwise refinement applicable with hypersafety

Hyperliveness? In general?

# Stepping Back...

- Safety and liveness? ✓
- Verification?
  - Model-checking (expensive) ✓
  - Reduce to trace properties ( $k$ -safety) ✓
  - Refinement (hypersafety) ✓
  - Proof system? (ongoing work with Hunter Goldstein)

...verify by decomposing to safety+liveness?

# Summary

Theory of hyperproperties :

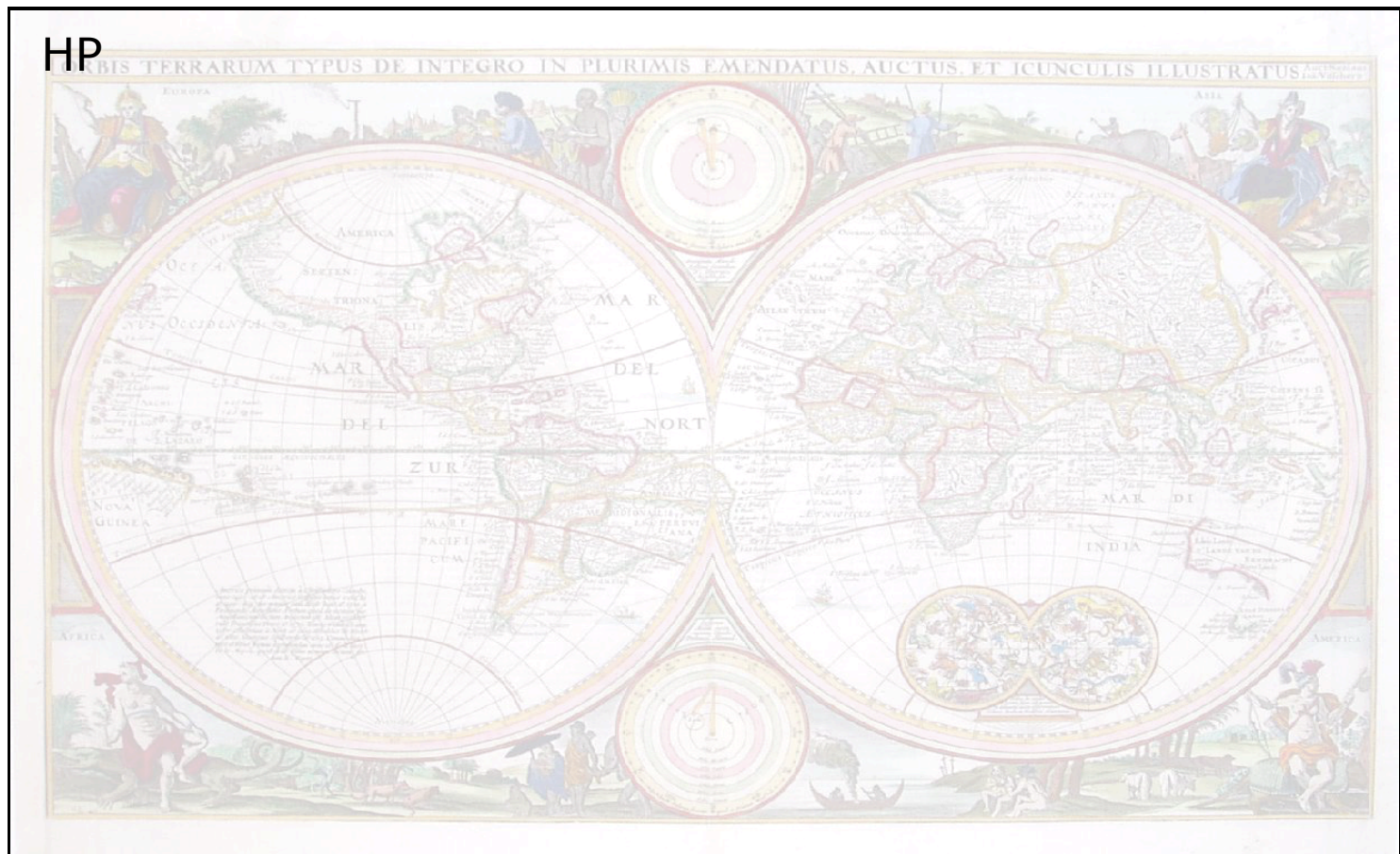
- Parallels theory of properties
  - Safety, liveness (basis, topological characterization)
  - Verification (HyperLTL,  $k$ -hypersafety, stepwise refinement)
- Expressive completeness
- Enables classification of security policies...



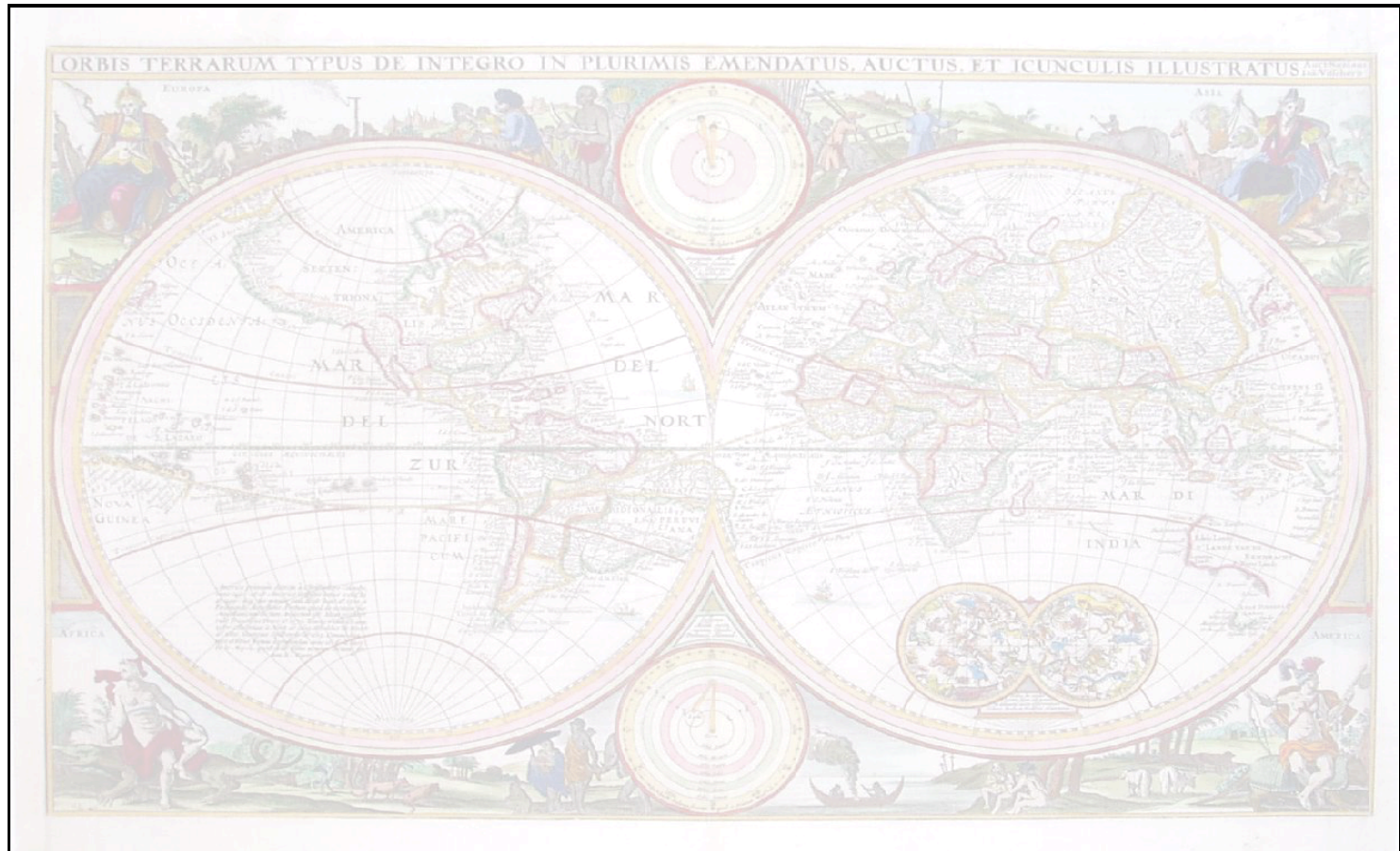
Charting the landscape...



HP

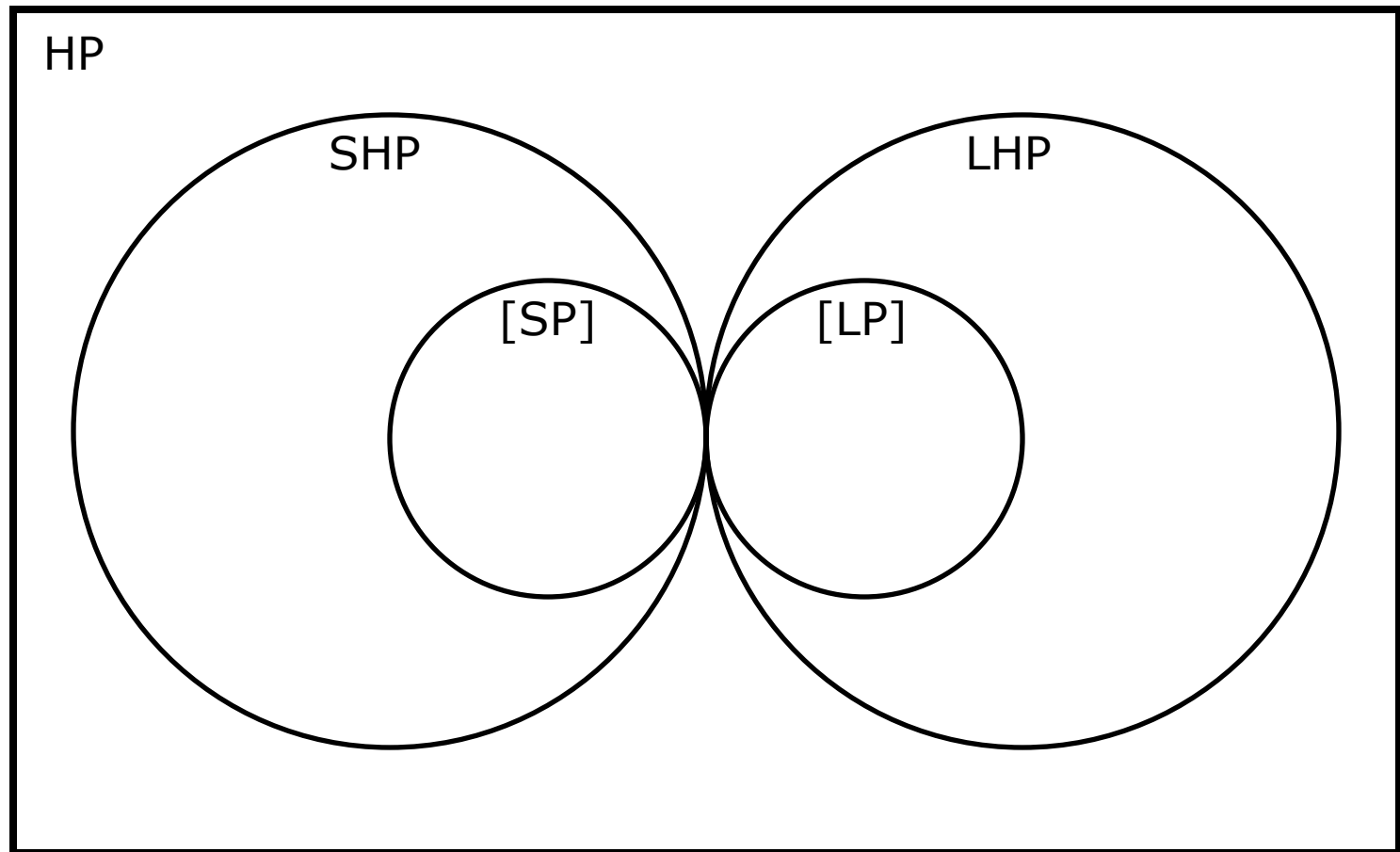


All hyperproperties (HP)

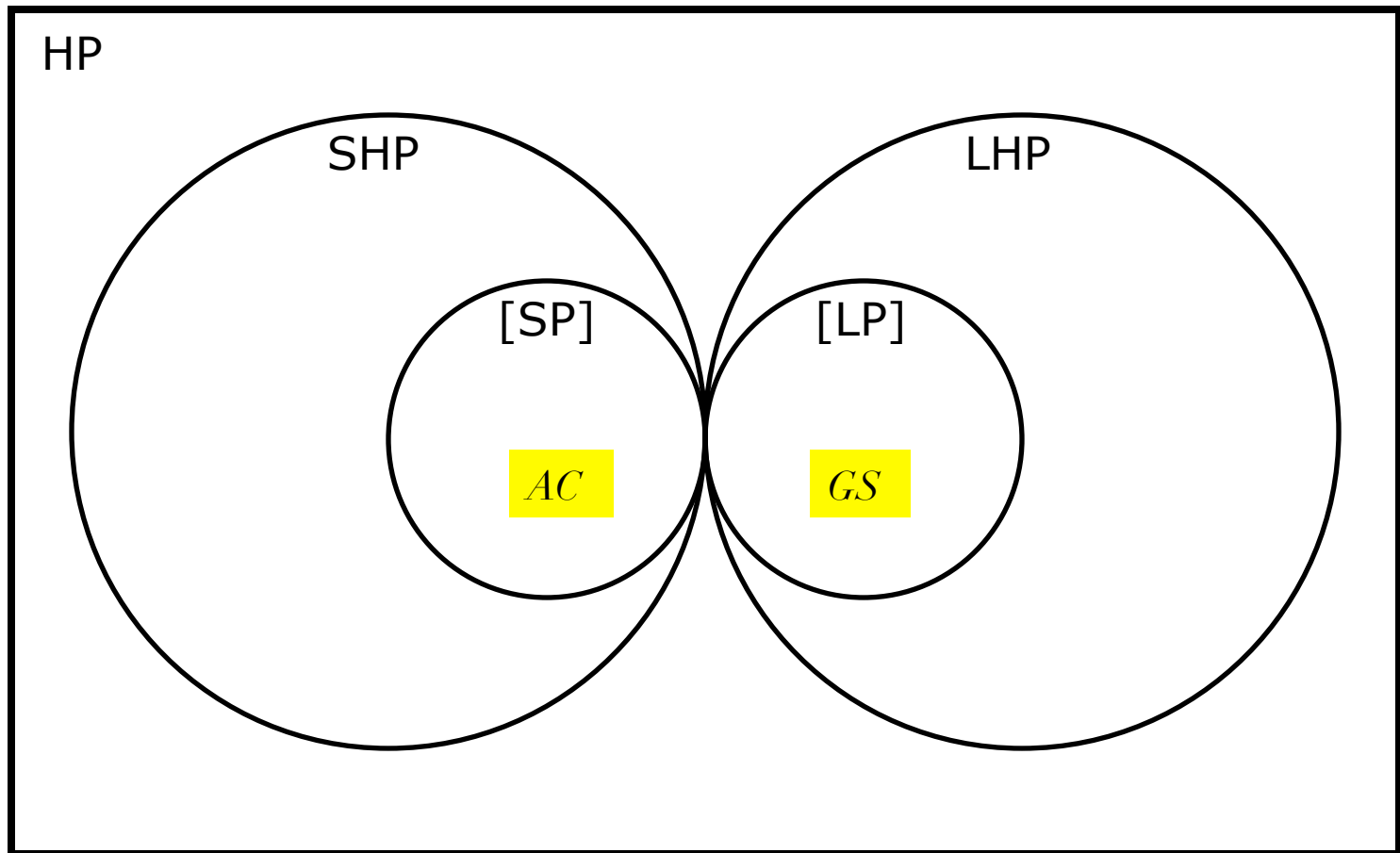


Safety hyperproperties (SHP)  
Liveness hyperproperties (LHP)

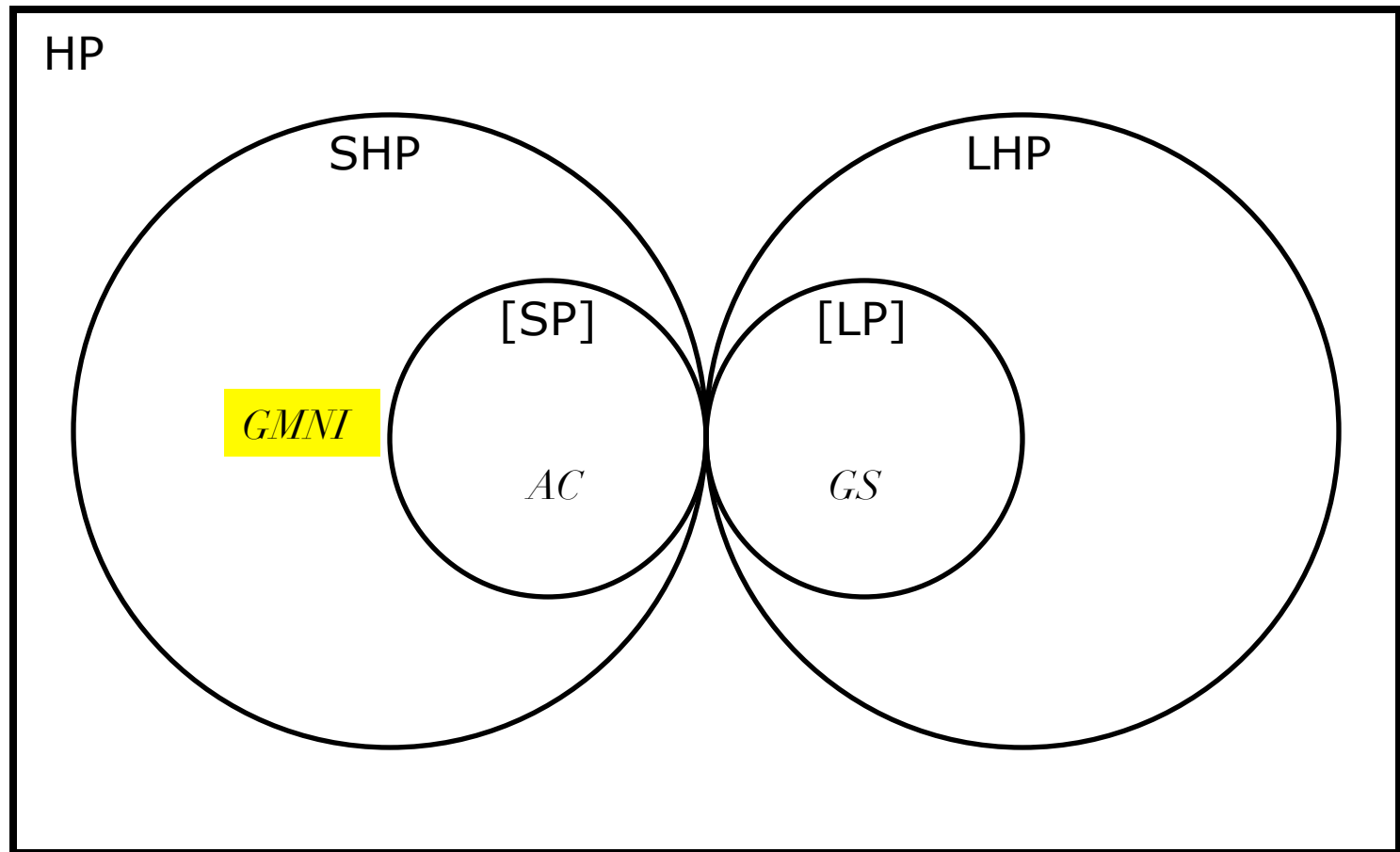




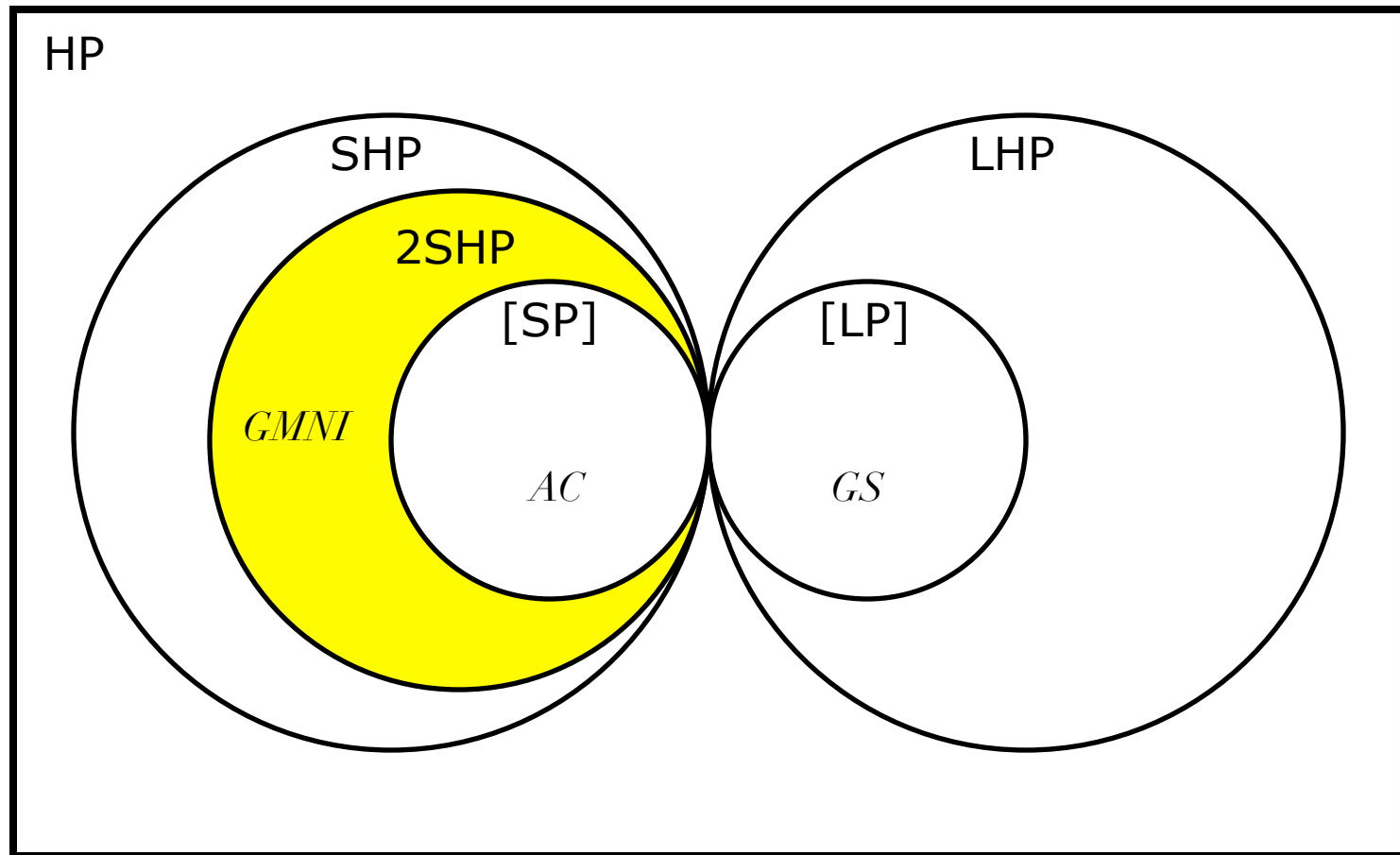
Lifted safety properties [SP]  
Lifted liveness properties [LP]



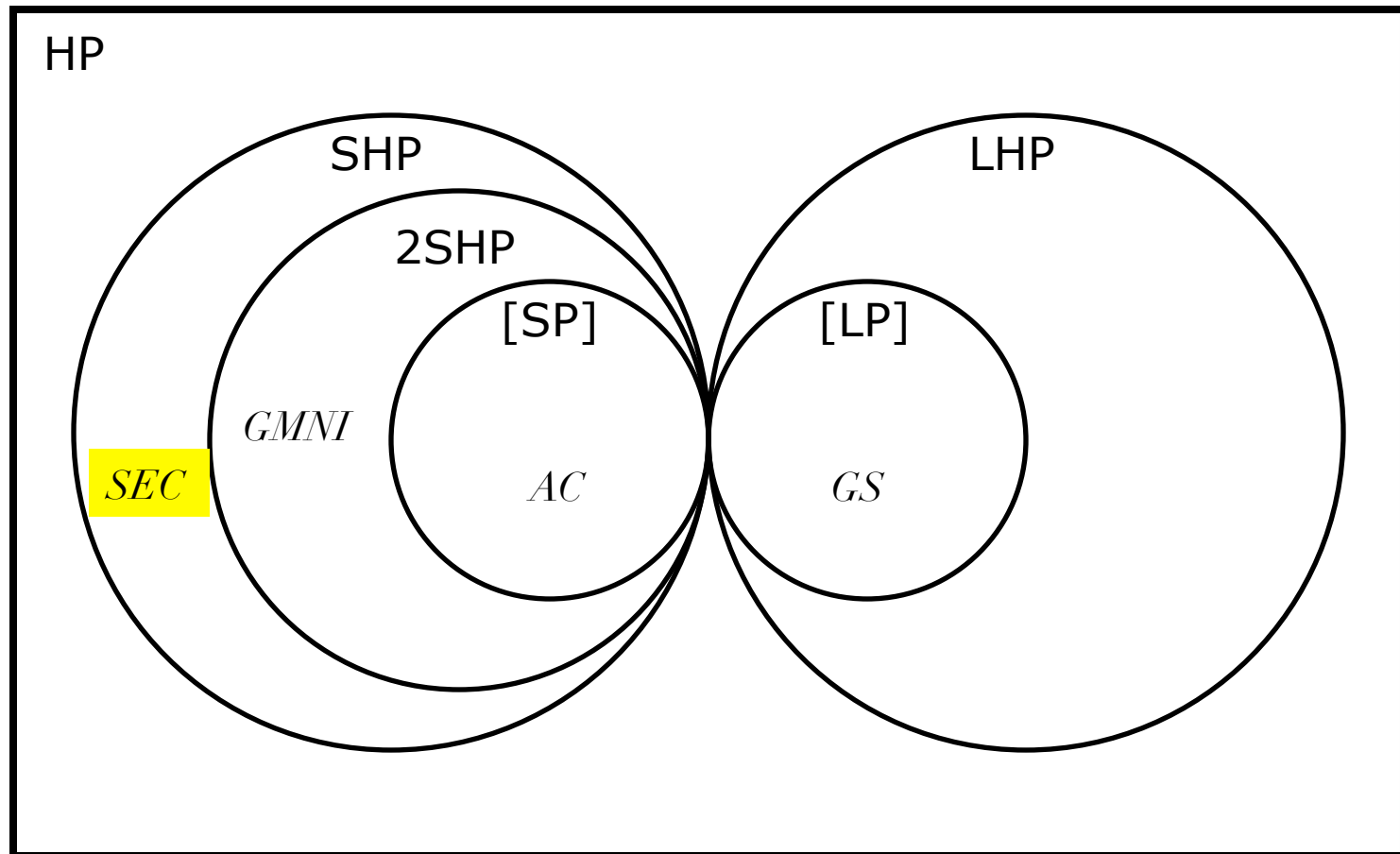
Access control ( $AC$ ) is safety  
Guaranteed service ( $GS$ ) is liveness



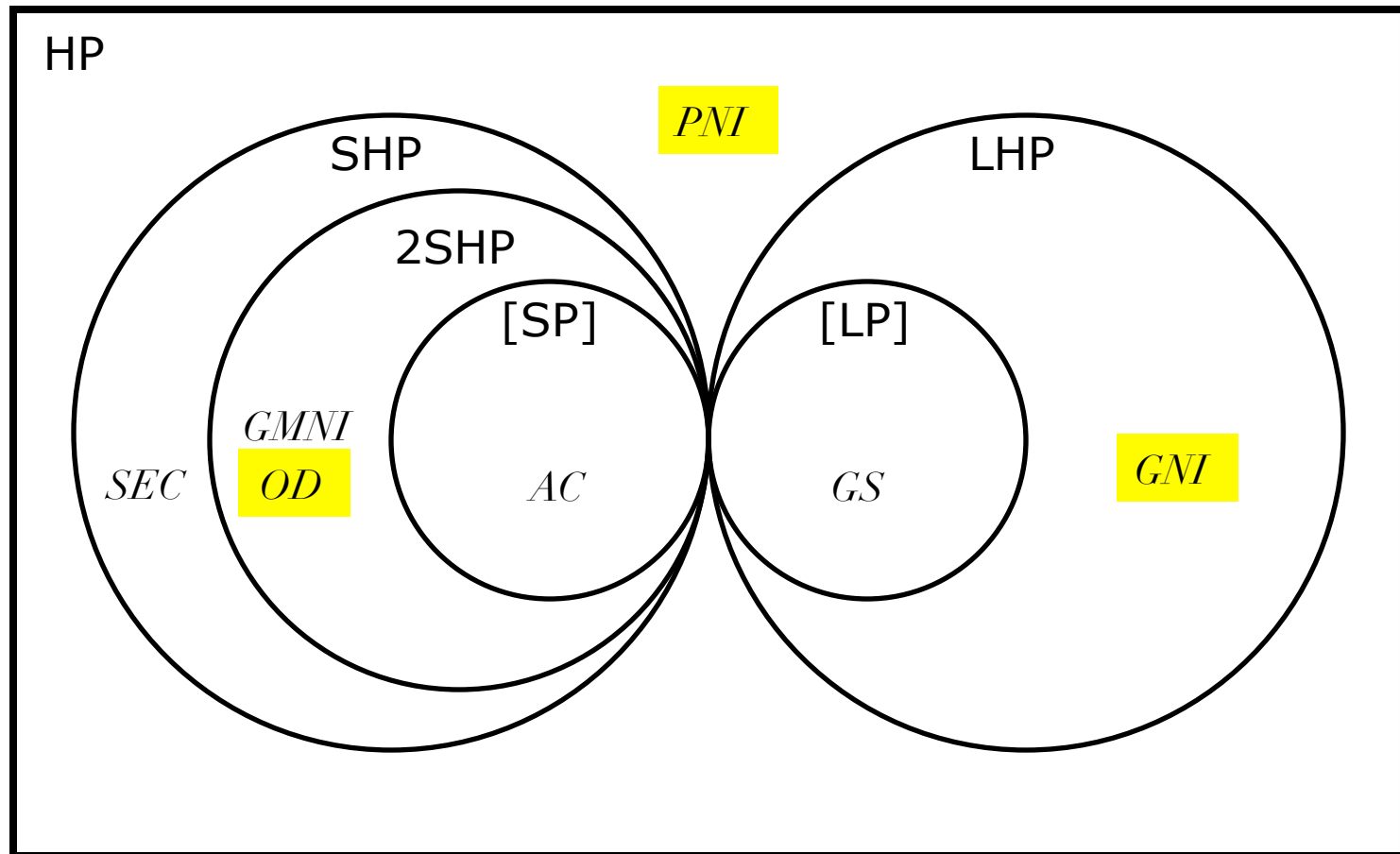
Goguen and Meseguer's noninterference (*GMNI*)  
is hypersafety



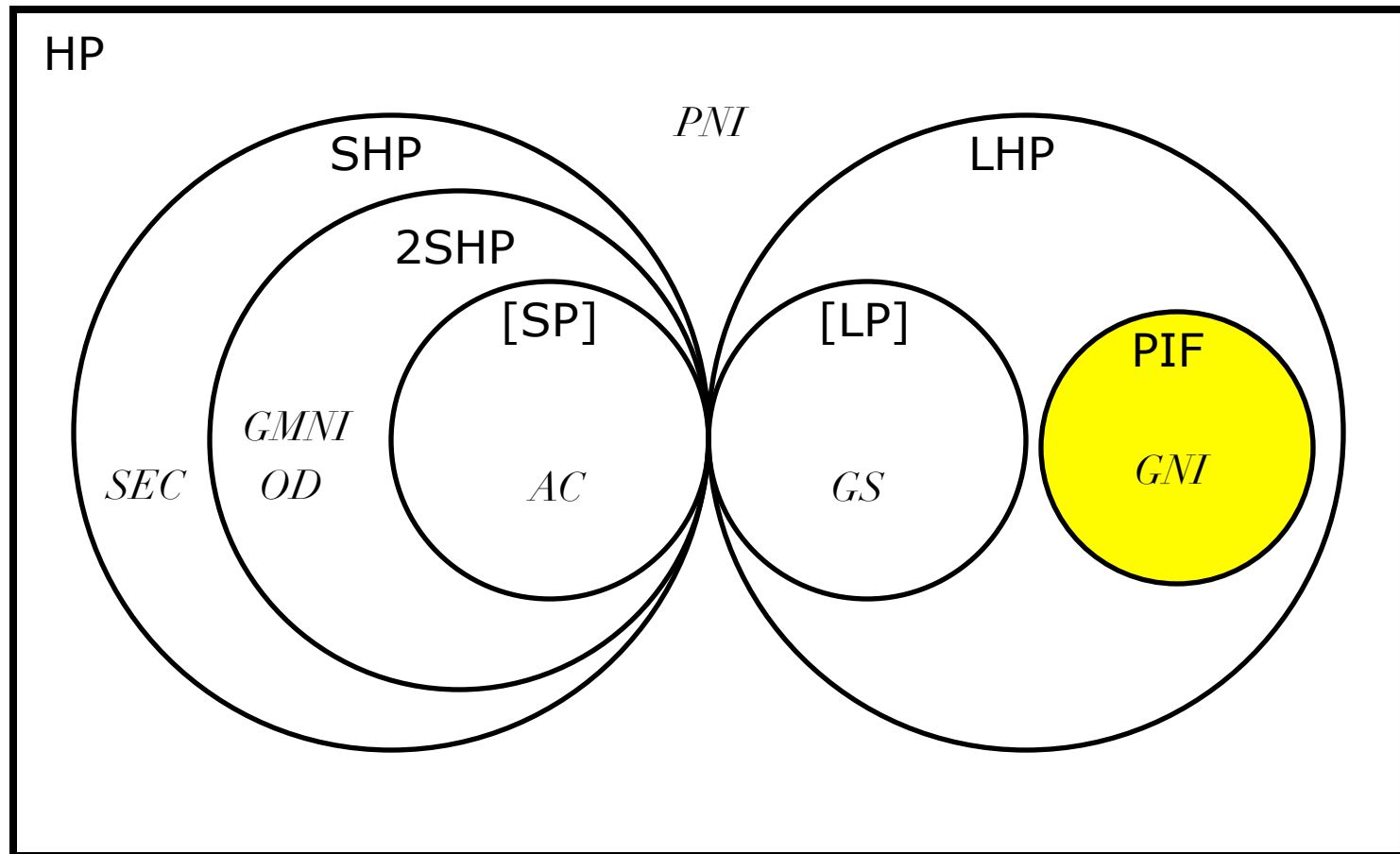
2-safety hyperproperties (2SHP)



Secret sharing (*SEC*) is not  $k$ -hypersafety for any  $k$



Observational determinism ( $OD$ ) is 2-hypersafety  
 Generalized noninterference ( $GNI$ ) is hyperliveness  
 Probabilistic noninterference ( $PNI$ ) is neither



Possibilistic information flow (PIF) is hyperliveness

# Revisiting the CIA Landscape

- **Confidentiality**

- Information flow is not a property
- Is a hyperproperty (HS: *OD*; HL: *GNI*)

- **Integrity**

- Safety property?
- Dual to confidentiality, thus hyperproperty?

- **Availability**

- Sometimes a property (max. response time)
- Sometimes a hyperproperty (HS: % uptime, HL: avg. resp. time)

➔ CIA seems unrelated to hyperproperties



# Reading

- **Hyperproperties.** *Journal of Computer Security* 18(6): 1157–1210, 2010. With Fred B. Schneider.
- **Temporal Logics for Hyperproperties.** In *Proc. POST*, pp. 265-284, 2015. With Bernd Finkbeiner, Masoud Koleini, Kristopher Micinski, Markus Rabe, and Cesar Sanchez.

# Upcoming events

- [May 16] Final exam

