Book and Article References for Cornell's CS-Math-Applied Logic Course

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Abstract

There are many excellent textbooks, reference books, history books and articles on the topics covered in Cornell's Math/CS Applied Logic course. Several of my favorites are listed and categorized in this document. The first item is the textbook *First-Order Logic* by Raymond Smullyan that I have frequently used for the course.

1 Logic Books

1. *First-Order Logic* [83].
2. *Raymond Smullyan on Self Reference* [45].
3. *Introduction to Metamathematics* [76].
4. *Foundations of Intuitionistic Mathematics* [77].
5. The Calculi of Lambda-Conversion [15].
6. *Introduction to Mathematical Logic* [16].
7. *Logic for Applications* [82].
8. *Edinburgh LCF: a mechanized logic of computation* [58].
9. *An Introduction to Mathematical Logic and Type Theory*: to Truth through Proof [1].
10. *The Language of First-Order Logic* [3].
11. *Essays in Constructive Mathematics* [35].
12. *Computability and Logic, Second Edition* [9].

13. *Computability and Logic, Third Edition* [8].
14. *A Modern Perspective on Type Theory: From its Origins until Today* [75].
15. *A Course in Mathematical Logic* [5].
16. *A Computational Logic* [11].
17. *A Computational Logic Handbook* [12].
18. *The Lambda Calculus: Its Syntax and Semantics* [2].
19. *Toposes and Local Set Theories* [6].
20. *Combinatory Logic, Volume I* [25].
21. *Combinatory Logic, Volume II* [26].
22. *Higher Order Logic Theorem Proving and its Applications* [17].
23. *Mathematical Logic* [14].
24. *Introduction to the PL/CV2 Programming Logic* [20].
25. *Model Theory* [13].
26. *A Programming Logic* [21].
27. *Mathematical Logic* [14].
28. *The Lambda Calculus: Its Syntax and Semantics* [2].
29. *The Structure of Nuprl's Type Theory* [18].
30. *Logic and Information* [29].
31. *The Millenium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time* [30].
32. *Logicomix: An Epic Search for Truth* [31].
33. *Mathematical Intuitionism: Introduction to Proof Theory* [32].
34. *Elements of Intuitionism* [33].
35. *Frege Philosophy of Mathematics* [34].
36. *Recursiveness* [37].
37. *Automata, Languages and Machines* [36].
38. *A Mathematical Introduction to Logic* [38].
39. *Formal Languages: Automata and Structures* [39].

40. *The Elements* [40].
41. *Foundations of Set Theory* [46].
42. *Reasoning About Knowledge* [41].
43. *Kurt Gödel Collected Works* [42].
44. *Symbolic Logic, An Introduction* [43].
45. *Intuitionistic model theory and forcing* [44].
46. *Foundations of Set Theory* [46].
47. *Begriffsschrift, A Formula Language, modeled Upon that for Arithmetic for Pure Thought* [47].
48. *Proofs and Types* [53].
49. *Semantical Investigations in Heyting's Intuitionistic Logic* [48].
50. *Logic for Computer Science, Foundations of Automatic Theorem Proving* [50].
51. *Constructive Logics. Part I: A Tutorial on Proof Systems and Typed λ-Calculi* [49].
52. *Simple Heuristics That Make Us Smart* [51].
53. *Proof Theory and Logical Complexity* [52].
54. *On Formally Undecidable Propositions of Principia Mathematica and Related Systems* [54].
55. *Recursive number theory* [55].
56. *Recursive Analysis* [56].
57. *Treatise on Intuitionistic Type Theory* [59].
58. *Theory of Program Structures: Schemes, Semantics, Verification* [60].
59. *Introduction to Combinatory Logic* [69].
60. *Logical Frameworks* [72].
61. *First Order Dynamic Logic* [61].
62. *Handbook of Practical Logic and Automated Reasoning* [63].
63. *The Logical Foundations of Mathematics* [64].
64. *On Intelligence* [65].
65. *Introduction to Mathematic Logic* [66].
66. *Intuitionism, An Introduction* [67].

67. *Foundations of Geometry* [68].

68. Gödel, Escher, Bach: *An Eternal Golden Braid* [71].

69. *Logic-Based Knowledge Representation* [73].

70. *Logic: Techniques of Formal Reasoning* [74].

2 Proof Assistants

The thirty five year steady increase in the effectiveness of *proof assistants* has brought them into classrooms, university research groups, and industrial labs around the globe. At Cornell we designed and built Nuprl in the period from 1985 until 1986 and have continued to use, extend and improve improve it ever since. The 1986 version is described in the book *Implementing Mathematics* [19]. We continue to extend the *Nuprl* proof assistant. We were inspired by *Automath* [27, 28, 81] and by the *Edinburgh LCF system* [58], and Sir Tony Hoare's work on data structures [70]. We believe that in due course modern proof assistants will exhibit a significant quantum increase in their *machine intelligence*, and that innovation will rapidly spread. We have seen evidence of this with Nuprl. Coming advances will help people build more reliable software with provably guaranteed properties. They will embolden mathematicians to attack more difficult open problems. Moreover proof assistants are a harbinger of something *broader and more impactfull* that we discuss later in this article. Continuing advances in proof assistant design and implementation will become "game changing." In this effort, the US and EU will continue their highly productive cooperation, dating back to de Bruijn's *Automath* [27], to the creation and deployment of of modern proof assistants, such as *Agda* [10], *Coq* [23, 22, 24, 7], *HOL* [57, 62], and *Nuprl* [4, 19, 78] among others.[1]

2.1 Modern Automated Reasoning and Formal Methods

The rich new type theory implemented by Nuprl in 1985 combined with the ground breaking ideas of de Bruijn [27, 28], McCarthy [79, 80], Milner [58] and others for automating reasoning *created a new subject area of computer science*. Research groups at major universities such as CMU, MIT, Princeton, UPenn, Edinburgh, Oxford, Cambridge, Potsdam, as well as at major research centers such as INRIA, Intel, Microsoft Research (MSR), and others, are advancing this important research. The events to monitor are those which help the proof assistant discover new results. We recently saw a glimpse of something exciting when Nuprl suggested a lemma that led Mark Bickford to a wonderful new result entitled *Connectedness of the Continuum in Intuitionistic Mathematics* See the Nuprl web page. The result will be published in *Mathematical Logic Quarterly* in 2019.

[1]There is no complete list of proof assistants, but a Wikipedia page on proof assistants lists 14 of them as of 2018. More are under construction.

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