(B1) $z \in x \to S(x)$.
(B2) *Cases*: $\mathbf{d}nnxy = x \wedge (n \neq m \to \mathbf{d}nmxy = y)$.
(B3) *Successor*: $N(0) \wedge N(\mathbf{s_N}n) \wedge (\mathbf{s_N}n = \mathbf{s_N}m \to n = m) \wedge \mathbf{s_N}n \neq 0$.
(B4) *Induction*: $\phi(0) \wedge \forall n(\phi(n) \to \phi(\mathbf{s_N}n)) \to \forall n\phi(n)$, all formulae ϕ.

Conventions. a, b, u, v, w are (meta)variables for sets; i, j, k, n, m are for numbers. These conventions have been used to abbreviate the axioms. Other variables such as x, y, z are unrestricted.

Remark. Note that the pairing axiom permits us to form a set $\mathbf{p}xy$, usually written $\{x, y\}$, from any two objects x and y, not only from two sets.

7. Axioms of set theory

In this section we list for reference the axioms of Zermelo–Fraenkel set theory. These axioms are formulated in a language with binary relation symbols $\in$ and $=$, and no other constants, relation, or function symbols.

Extensionality: $\forall x(x \in a \leftrightarrow x \in b) \to a = b$.
Pairing: $\exists a(x \in a \wedge y \in a)$.
Union: $\exists a \forall x(x \in a \leftrightarrow \exists b \in a(x \in b))$.
Separation: $\exists a \forall x(x \in a \leftrightarrow x \in b \wedge \phi)$ (a not free in ϕ).
Infinity: $\exists a(\exists x \in a \wedge \forall x \in a \exists y \in a(x \in y))$.
Powerset: $\exists a(\forall z(z \in x \to z \in b) \to x \in a)$.
$\in$-induction: $\forall u(\forall x \in u\phi(x) \to \phi(u)) \to \forall u\phi(u)$.
Empty set: $\exists a \forall x(x \notin a)$.
Replacement: $\forall x \in a \exists! y\phi \to \exists b \forall x \in a \exists y \in b\phi$.
Collection: $\forall x \in a \exists y\phi \to \exists b \forall x \in a \exists y \in b\phi$.

These are the versions of the axioms which Friedman [18] discovered were suitable for use with intuitionistic logic; note that they are the same as the usual axioms except that the axiom of foundation has been replaced by $\in$-induction. Also, the axioms of replacement and collection are equivalent if classical logic is allowed, but not with intuitionistic logic [21]. We therefore have the following set theories:
- **IZF-Rep**: all of the above axioms except collection, with intuitionistic logic,
- **IZF-Col**: all of the above axioms, with intuitionistic logic,
- **ZF**: all of the above axioms, with classical logic.

Of course, collection implies replacement, so there is no need to include replacement on the list of axioms of **ZF** or **IZF-Col**.

This paper can be understood without knowing anything about intuitionistic logic, but it is intuitionistic logic that would be built into a computation system based on **IZFR**, and so a background in intuitionistic set theory is relevant. See [4, Chapter VIII].