Course Project Guidelines

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Abstract

This note discusses course projects and the grading criteria for CS4860 Fall 2019. Project papers are due on the last day of classes, December 10.

1 Information about Projects and Grading

Grades will be based on a short paper that focuses on a topic in logic of mutual interest to the student and me. The submitted paper should be related to the material covered in lectures and in suggested readings. Topics that connect type theory to other concerns of computer science and mathematics are especially relevant. Connections to Brouwer and intuitionism are of special interest.

Another possible theme is the treatment of the real numbers, $\Re$, in constructive and intuitionistic analysis. Dr. Bickford has implemented in Nuprl several results from the book by Bishop and Bridges *Constructive Analysis* [1]. We will not be able to devote more than two lectures on this topic. However, interested students can find a connection between the book and the Nuprl work on constructive real analysis at this url: www.nuprl.org/MathLibrary/ConstructiveAnalysis/

The role of proof assistants in mathematics and computer science is especially relevant. Computational Euclidean geometry is a topic of special interest since our research group has done important work in this area [2]. Brouwer's fixed point theorem is another topic of special interest to my colleagues and me. Writing a report on the Nuprl implementation of this famous theorem is an interesting topic for an applied logic course.

The intuitionistic type theory we mentioned in the course was made possible by the foundational results of Kleene and Vesley in their ground breaking book *The Foundations of Intuitionistic*

Mathematics [3] and Kleene's earlier results on intuitionistic number theory. Kleene visited Brouwer in Holland and exchanged insights with him.

We will relate this new work to our implementation of the long standing constructive treatment of the Bishop and Bridges book *Constructive Analysis* [1] in Nuprl with a new account of many of the ideas and results using the fully intuitionistic type theory.

The writings of Per Martin-Löf [4] cover part of the range of intuitionistic type theory. Writing a report that compares his account of type theory to what we covered in the course would be quite educational and interesting.

2 Progress to a fully intuitionistic type theory

Here are four articles that report on our implementation of a fully intuitionistic type theory. An interesting project would be relating one of these articles to topics in the course. The articles can be downloaded from the Nuprl web page.

- Bar Induction: The Good, the Bad, and the Ugly
- Computability Beyond Church-Turing via Choice Sequences
- Validating Brouwer's Continuity Principle for Numbers using Named Exceptions
- Bar Induction is Compatible with Constructive Type Theory

References

[1] E. Bishop and D. Bridges. *Constructive Analysis*. Springer, New York, 1985.

[2] Ariel Kellison, Mark Bickford, and Robert Constable. Implementing Euclid's straightedge and compass constructions in type theory. *Annals of Mathematics and Artificial Intelligence*, Sep 2018.

[3] S. C. Kleene and R. E. Vesley. *Foundations of Intuitionistic Mathematics*. North-Holland, 1965.

[4] Per Martin-Löf. Constructive mathematics and computer programming. In *Proceedings of the Sixth International Congress for Logic, Methodology, and Philosophy of Science*, pages 153–175, Amsterdam, 1982. North Holland.