CS4860 Oct 17, 2019 Lecture

Axioms for Zermelo-Fraenkel set theory
(from Thomas Jech Set Theory, Academic Press 1978)

I Extensionality If X and Y have the same elements, then $X = Y$.

II Pairing For any a, b there is a set $\{a, b\}$

III Separation Let φ be a property (with parameter p) then $Y = \{u \in X \mid \varphi(u, p\}$ is a set

IV Union $Y = \cup X$ for any set X is the union of all elements of X

V Power set $P(X)$ the set of all subsets of X

VI Infinity There is an infinite set.

VII Replacement $\{F(x) \mid x \in X\}$ for F a function.

VIII Regularity Every non empty set has an $\in$-minimal element.

IX Choice For every family of non-empty sets, we can pick an element of each set and collect them into a set.

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ZFC Axioms (Just & Weese)

A1 Extensionality $\forall x,y.(x = y \Leftrightarrow \forall z.(z \in x \Leftrightarrow z \in y))$

A2 Empty set $\exists x \forall y (y \notin x)$

A3 Foundation $\forall x (x \neq \emptyset \Rightarrow \exists y \in x. \forall z \in x.(z \notin y))$

A4 Pairing $\forall x,y. \exists z. \forall u.(u \in z \Leftrightarrow (u = x \vee u = y))$

A5 Union $\forall x \exists y \forall z (z \in y \Leftrightarrow \exists u (u \in x \,\&\, z \in u))$.

A6. Comprehension Schema $\forall z \exists y \forall x.(x \in y \Leftrightarrow x \in z \,\&\, \phi(x))$

A7. Replacement $\forall a (\forall x \in a. \exists! y\, \phi(x,y) \Rightarrow \exists z \forall x \in a. \exists y \in z. \phi(x,y))$

A8. Power set $\forall x\, \exists y. \forall z.(z \in y \Leftrightarrow z \subseteq x)$

A9. Choice For every family of non-empty, pairwise disjoint sets, there is a set z $|z \cap y| = 1$ for each $y \in x$

A.10 Infinity $\exists x.(x \neq \emptyset \,\&\, \forall u (u \in x) \Rightarrow u \cup \{u\} \in x))$

Beeson's Axioms – see on-line lecture notes

A1 Extensionality

A2 Pairing

A3 Union

A4 Separation

A5 Infinity

A6 Powerset

A7 $\in$-induction

A8 Empty set

A9 Replacement

A10 Collection