Abstract

These notes are comments on Brouwer’s Cambridge Lectures on Intuitionism [2]. They summarize his points in terms of concepts we will cover in the fall 2019 version of CS/Math 4860. Some of the results will be related to the Kleene and Vesley book on intuitionism [3] and to the Cornell research into intuitionism [1].

1 Historical Introduction and fundamental notions

The title of this section is the title of Brouwer’s first section. It is about the origin and grounds for mathematical certainty and the limits of mathematical science. He says that mathematics is grounded in our intuition of time and space, with time being the most fundamental. Already on the first page we can see why his approach to mathematics is called intuitionism.

In the same section he also notes that as the fundamental properties of space and time were investigated, the most fundamental ones were taken to be axioms and expressed in language. As more and more complicated properties were investigated, the most familiar and exact were called axioms. As the topics investigated became more complex, the clearest concepts and principles were taken as axioms, and the means of deduction were based on logic. (I would add that this approach was already well developed in Euclid and his point was easily grasped.)

The use of deduction and axioms made mathematics dependent on logic. I would add that we can see that dependence clearly in Euclidean geometry. It was concerned with space, and its deductive structure was based on logic. Brouwer calls this method the observational standpoint.
Next he says that this observational standpoint became untenable in the 19th century as a result of discoveries by Riemann, Klein, Hilbert, Einstein and others. He picks quite a distinguished list of mathematicians and physicists. He calls their method of working the logico-linguistic method. It used words and logical rules. Its main proponents were Dedekind, Cantor, Peano, Hilbert, Russell, Zermelo, and Couturat, a major French champion of symbolic logic.

Next Brouwer mentions the Pre-intuitionists, Poincarè, Borel, and Lebesgue. He says that they had an impoverished view of the continuum. This led them to think that ordinary classical logic was sufficient to understand the continuum. In part because this logic used the famous law of excluded middle, it was not faithful to experience with the continuum. He notes that Hilbert stressed the study of meta-mathematics, the study of the logical systems for understanding logic.

2 FIRST ACT OF INTUITIONISM

Brouwer says that the first act of intuitionism is completely separating mathematics from mathematical language and hence from logic. He says that intuitionistic mathematics is an essentially language-less activity of the human mind having its origins in the perception of time.

He goes on to say that the principle of excluded third, also called the law of excluded middle, is not valid in thinking about his mathematics. He says that mathematical language by itself can never create new mathematical systems.

I would surmise that logical primitives such as \&, \lor, \neg, \Rightarrow actually arose out of experience in dealing with evidence. I will make this case during the course as we explore logical principles in detail.

References

