Problem Set 4

CS 4860 - Spring 2018

March 26, 2018

1. Translate the following PC propositions into iPC using the double negation transform shown in lecture and in the reading from Kleene for Lecture 18.
   - Pierce's Law: $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$
   - Smullyan p.24: $\neg(P \ \& \ Q) \Rightarrow (\neg P \ \vee \neg Q)$
   - $(P \Rightarrow Q) \Rightarrow (\neg P \ \vee \ Q)$
   - $(P \ \& \ Q) \Rightarrow \neg(P \Rightarrow \neg Q)$
2. Translate the following FOL theorem in iFOL using Kleene's $A^{0\dagger}$ conversion:

   $$((\forall x : D.\ A(x)) \Rightarrow B) \Rightarrow \exists x : D.\ (A(x)) \Rightarrow B)$$

   Note, the A^0 conversion gives

   $$((\forall x : D.\ A(x)) \Rightarrow B) \Rightarrow \neg \forall x : D.\ \neg(A(x) \Rightarrow B)$$

   The $A^\dagger$ conversion replaces each prime part P by $\neg\neg P$.
3. Write an informal proof, using Kleene's axioms for Lecture 9, that $x + y = y + x$. Frist give and intuitive account of this result.
4. Give an informal proof of Euclid's theorem that there are an unbounded number of primes. Translate this into a proof in informal Heyting Arithmetic (HA) using Kleene's induction axiom (from Lecture 9).