

CS4860 - Problem Set 3

Due Tuesday, March 13th

March 1, 2018

Please read Smullyan pages 43-55 this week.

There will be two more problems added by March 1st.

1. Solve Exercise 1 page 51. Add an explanation of why

$$\forall x. (P(x) \& Q(x)) \Rightarrow \forall x. P(x)$$

is logically true but not a tautology.

2. Prove the first five Exercises on page 56 of Smullyan.
3. Kleene's axioms are sufficient to prove all classically valid first-order formulas. Show how to prove the following using his axioms.

$$\forall x. (P(x) \& Q(x)) \Rightarrow \forall x. P(x) \& \forall x. Q(x)$$

4. Think about this example from Smullyan p.56

$$\exists y. (P(y) \Rightarrow \forall x.P(x)).$$

Why is it intuitively true? Is it constructively valid?

1. Please read Smullyan pages 43 to 56 in Part II.
2. Give a Tableaux proof of the following two propositions in the Exercises on page 56 under the heading Exercises Prove the following two formulas which are the first example and the third example under Exercises on page 56.
 - (a) $\forall y. [\forall x. P(x) \Rightarrow P(y)]$
 - (b) $\neg \exists y. P(y) \Rightarrow [\forall y. (\exists x. P(x) \Rightarrow P(y))]$
3. Give the evidence semantics for (a).