Problem Set 3 and iPC Decision Procedures

October 18, 2018

1 Theme

We have examined Fitting's approach to explaining the meaning of iPC formulas using *Beth semantics*, a variant of *Kripke semantics*. There does not seem to be a result showing that if a formula of iPC is false in this Beth/Kripke semantics then we cannot write a program for it. Mark, Ariel, and I have been trying to prove such a result for class, and we might in due course succeed, but our approach will depend on knowing more about type theory.

This investigation of iPC shows that constructive type theory is a more suitable context for studying the semantics of iPC than the Beth/Kripke models. However this presents a pedagogical dilemma because we are using iPC to introduce basic ideas of constructive type theory.

Here are five very simple homework problems related to this study. Please write up solutions for next Tuesday October 23.

1. We analyzed the proposition $\neg(A \wedge B) \Rightarrow (\neg A \vee \neg B)$ and convinced ourselves that it is not programmable. For this exercise, apply Fitting's proof rules from page 28 of his book, already supplied as a handout, to show that this proposition is not provable. What does this tell us about programmability?

2. Can we show that a proposition A is not programmable by showing that $\neg A$ is provable? Give an example.

3. Analyze the proposition $\neg\neg(A \vee \neg A)$ by giving a program for it. Also give the very short and clear informal proof that this is constructively true. Recall Brouwer's very succinct informal proof of this fact.

4. Is this formula programmable? $(A \Rightarrow \neg B) \Rightarrow \neg(A \wedge B)$

5. Is the above formula provable using Fittings rules?

   How all of this logic ties into the larger story of life is illustrated by George B. Dyson's book, *Darwin Among the Machines*, Addison-Wesley, 1997. In the Preface he says:"In the game of life and evolution there are three players at the table: human beings, nature, and machines. I am firmly on the side of nature. But nature, I suspect, is on the side of machines." This quote will become more relevant as we study proof assistants and their connection to AI.