CS/Math-4860 Lectures 3 and Smullyan's Tableaux Proofs

September 4, 2018

Abstract

Lecture 3 covers Smullyan First-Order Logic Chapters I and II. The goal is to explain tableaux proofs by elaborating slightly what Smullyan has written.

Lecture 4 covers chapter 3 on *compactness* and König's Lemma. This is the first mathematically deep result we are exploring. We will come back to this topic when we look at computational interpretations of logic.

1 Tableaux Systems for Logic

The idea behind a tableaux proof is that we systematically search for a way to find an assignments of truth values to the propositional variables that will falsify the formula. If all attempts fail, Smullyan proves that the formula must be a tautology, i.e. it has the truth value T for all possible assignments of T and F to the propositional variables. Of course if an attempt to falsify succeeds, then we have direct evidence that the formula is not a tautology. This is a simple and powerful idea that can be extended to full first-order logic by showing how to treat the two quantifiers, for all, $\forall$ and $\exists$.

2 Compactness

We will examine Smullyan's proof of compactness on page 33. It uses König's Lemma. These ideas will be important when we study the completeness of First-Order Logic, a main result of the course.