1 Notes on Smullyan

Definition of an unordered tree (page 3):
1. A set of elements called points, S.
2. A function $F$ which assigns to each point $x$ a positive natural number (in $\mathbb{N}^+$) called the level of $x$.
3. A relation $x R y$, read “$x$ is the predecessor of $y,”$ such that
   - there is a unique point of level 1 called the origin (or root).
   - every point other than the origin has a unique predecessor.
   - for all $x, y$, if $y$ is the successor of $x$, then $F(y) = F(x) + 1$.

We also define:
- endpoint = no successor
- simple point = one successor
- junction point = more than one successor
- path

- A tree is called finitely generated (p.4) if each point has only finitely many successors (also see p.31).
- A tree is finite if it has finitely many points.
- A finitely generated tree can be infinite (unbounded).

Question (page 31) : If for every level $n$ there is at least one point at level $n$, does the tree have an (infinite) unbounded length path?

2 Related Lemmas and Theorems

- König’s Lemma: Every finitely generated tree $T$ with infinitely many points has at least one infinite branch.
- Fan Theorem: Every finitely generated tree $T$ in which every path is bounded ($\forall \alpha : \text{Path}(T). \exists n : \mathbb{N}. |\alpha| < n.$) is finite.