Lecture 6

CS 4860

September 8, 2016

1 Introduction

The Tableaux style logic is also good as a logic for finding a program in a polymorphic type specification. We call this a *function programming logic*. The logic arises when we reveal the computational features of the rules. This is done by showing the program elements that correspond to each rule. We follow the order of rules given by Smullyan on page 17.

We give the rules "names" that correspond to the programming construct. This is clearest for the "implication right" rule. Another name for it is the *function introduction rule*. We give a simple application of it first.

$$\vdash X \to X \;\; by \;\; \lambda(x.\_\_\_)$$
$$x : X \vdash \;\; by \;\; x$$

The general rule is rule (4) in Smullyan and the handout.
Here it is:

$$h_1 : H \vdash X \to X \;\; by \;\; \lambda(x.\_\_\_)$$
$$h_1 : H,\; x : X \vdash X \;\; by \;\; x$$

2 Refinement Rules with Realizers (code)

1. Functions (implication)
   Construction(Right)
   $$h : H \vdash X \to Y \;\; by \;\; \lambda(x.\_\_\_)$$
   $$h : H,\; x : X \vdash Y \;\; by \;\; exp(h, x)$$
   Application (Left)
   $$h_1 : H,\; f :\; X \to Y, h_2 : H_2 \;\vdash G \;\; by \;\; ap(f; \_\_\_),\; z = ap(f; \_\_\_)$$
   $$\vdash X \;\; by \;\; a(h_1)$$
   $$z : Y \vdash G$$

2. Pairs (conjugation)

   Construction

   $h : H \vdash X \;\&\; Y \;\; by \;\; pair(\_\_\_;\_\_\_)$

   " $\quad$ " $\vdash X \;\; left(h)$

   " $\quad$ " $\vdash Y \;\; right(h)$

   Decomposition

   $h_1 : H_1,\; a :\; X \;\&\; Y,\; h_2 : H_2 \vdash G \;\; by \;\; spread(a; x, y.\_\_\_)$

   $h_1 : H_1,\; x : X,\; y : Y,\; h_2 : H_2 \;\vdash G \;\; by \;\; g(h_1, x, y, h_2)$

3. Disjunction (variants)

   Construction

   $h : H \vdash X \;\vee\; Y \;\; by \;\; inl(\;\;)$

   $h : H \vdash X \;\; by \;\; exp_1(h)$

   $h : H \vdash X \;\vee\; Y \;\; by \;\; inr(\;\;)$

   $h : H \vdash \;Y \;\; by \;\; exp_2(h)$

   Deconstruction

   $h_1 : H_1,\; d :\; X \;\vee\; Y,\; h_2 : H_2 \vdash \;\; by \;\; decide(d; x.\_\_\_; y.\_\_\_)$

   " " $x : X,$ " " $\;\vdash G \;\; by \;\; g_l(h_1, x)$

   " " $y : Y,$ " " $\;\vdash G \;\; by \;\; g_r(h_1, y)$

4. Ex Falso

   $h_1 : H_1,\; x : \bot,\; h_2 : H_2 \;\vdash G \;\; by \;\; any(x)$