CS4860 Fall 2012 Problem Set 4

Due Tuesday, October 23, 2012

1. Propose one or two *project topics* with enough detail for us to evaluate them.

2. Provide evidence terms for these formulas from Smullyan page 56:

   - $\forall y.(\forall x.P(x) \Rightarrow P(y))$
   - $\exists x.(P(x) \vee Q(x)) \Rightarrow \exists x.P(x) \vee \exists x.Q(x)$
   - $\exists x.(P(x) \vee Q(x)) \Leftarrow \exists x.P(x) \vee \exists x.Q(x)$

3. Explain why there is no evidence for $(\exists x.P(x) \wedge \exists x.Q(x)) \Rightarrow \exists x.(P(x) \wedge Q(x))$.

4. Prove $(\exists x.Q(x) \wedge \forall x.P(x)) \Rightarrow \exists x.P(x)$ in Refinement Logic and in Tableau.

5. Give an argument that if there is evidence for a formula F in a finite domain $D = \{d_1, \ldots, d_n\}$ under the assumption that every atomic predicate $R(x, y)$ is decidable, e.g. $\forall x, y.(R(x, y) \vee \sim R(x, y))$, then F is truth functionally satisfiable.

6. Here is a proposition that is *truth functionally valid*, the question is what is the lowest dimension for the constructive proof?

   * $(\exists z.I(z) \wedge (\forall x.(I(x) \wedge Q(x) \supset \exists y.R(y))) \wedge (\forall x.(I(x) \wedge \sim Q(x) \supset \exists y.R(y)))) \supset \exists y.R(y)$

   (a) Explain why an instance (at least one perhaps more) of $P \vee \sim P$ is needed to provide evidence.

(b) Add the necessary evidence among the hypotheses by adding to one or more of the three hypotheses. Do this as economically as possible, e.g. make *the smallest addition that is sufficient*.

(c) Provide the evidence term for the augmented formula and discuss the dimension of this evidence based on our definition of dimension for IPC formulas.

7. *Show* $(H \wedge K) \supset L$, where

$H = (\forall x)(\forall y)[Rxy \supset Ryx]$ (R is *symmetric*)

$K = (\forall x)(\forall y)(\forall z)[(Rxy \wedge Ryz) \supset Rxz]$ (R is *transitive*)

$L = (\forall x)(\forall y)[Rxy \supset Rxx]$ (R is *reflexive* on its domain of definition).