Applied Logic	**Homework 3**
CS 4860 Fall 2012	**Due Date: Thursday, October 4, 5PM**

Homework 3

Reading Please read Smullyan p. 25-36, Chpt IV pages 43-52.

(1) Given an informal but informative proof that Refinement Logic for IPC is consistent for evidence semantics. Comment on why this is of practical value. (See Problem 4 for inspiration).

(2) Explain why the completeness proof for tableau (Thm 2, p. 28) is not a completeness theorem for IPC.

(3) Prove these theorems using Refinement Logic and create the evidence terms from the proofs. Explain why they are *uniform*.

  (a) $\sim\sim (P \vee \sim P)$

  (b) $(\sim P \vee Q) \Rightarrow (P \Rightarrow Q)$

  (c) $((P \Rightarrow (Q \vee R)) \wedge (P \wedge \sim Q)) \Rightarrow R$

(4) Execute the evidence terms from (3) on the following data. For (a) let P be *Goldbach's conjecture*, call it GC. Note that we have no evidence for either GC nor $\sim GC$.
For (b) let P be $(0 = 1)$ and Q be GC.
For (c) let P be $Prime(5)$ with p as the proof term,
let Q be $Even(5)$ and R be $Odd(5)$ with r as proof term.
Note, executing $\lambda x.x_2$ for $Prime(5) \wedge Odd(5) \Rightarrow Odd(5)$ is $\lambda x.x_2(< p, r >) = r$.

(5) Explain why König's Lemma (p. 32 of Smullyan) is not constructively true.

(6) * Solve the exercise on page 36 of Smullyan.
Hint, use this variant of König's Lemma:

Brouwer's Fan Theorem
If every branch of a finitely generated tree (say binary) is finite, then the tree is finite (finite number of points). This variant is intuitionistically true.

Extra Credit: Write up a proof of the *soundness* of tableau proofs (Smullyan p. 25) using your own ideas. Try to draw on the lectures as well as the reading.