Homework 3

Reading Please read Smullyan p. 25-36, Chpt IV pages 43-52.

(1) Given an informal but informative proof that Refinement Logic for IPC is consistent for evidence semantics. Comment on why this is of practical value. (See Problem 4 for inspiration).

(2) Explain why the completeness proof for tableau (Thm 2, p. 28) is not a completeness theorem for IPC.

(3) Prove these theorems using Refinement Logic and create the evidence terms from the proofs. Explain why they are uniform.

(a) \( \sim \sim (P \lor \sim P) \)
(b) \( (\sim P \lor Q) \Rightarrow (P \Rightarrow Q) \)
(c) \( ((P \Rightarrow (Q \lor R)) \land (P \land \sim Q)) \Rightarrow R \)

(4) Execute the evidence terms from (3) on the following data. For (a) let \( P \) be Goldbach’s conjecture, call it GC. Note that we have no evidence for either GC nor \( \sim GC \).
For (b) let \( P \) be \( 0 = 1 \) and \( Q \) be GC.
For (c) let \( P \) be Prime(5) with \( p \) as the proof term,
let \( Q \) be Even(5) and \( R \) be Odd(5) with \( r \) as proof term.
Note, executing \( \land x.x_x \) for \( \text{Prime}(5) \land \text{Odd}(5) \Rightarrow \text{Odd}(5) \) is \( \lambda x.x_x(< p, r >) = r \).

(5) Explain why König’s Lemma (p. 32 of Smullyan) is not constructively true.

(6) * Solve the exercise on page 36 of Smullyan.
Hint, use this variant of König’s Lemma:

Brouwer’s Fan Theorem
If every branch of a finitely generated tree (say binary) is finite, then the tree is finite (finite number of points). This variant is intuitionistically true.

Extra Credit: Write up a proof of the soundness of tableau proofs (Smullyan p. 25) using your own ideas. Try to draw on the lectures as well as the reading.