CS 4860 – Homework 1

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1.

(a) Let P_0 mean wages are raised, P_1 mean prices are raised, P_2 mean there is inflation, P_3 mean Congress regulates inflation, P_4 mean the people suffer, and P_5 mean the congressmen are unpopular. Then,

$$(((((P_0 \vee P_1) \implies P_2) \wedge (P_2 \implies (P_3 \vee P_4))) \wedge (P_4 \implies P_5)) \wedge (\neg P_3 \wedge \neg P_5)) \implies \neg P_0$$

(b) Let P_0 mean Jones wins the lawsuit, P_1 mean Rogers is entered into the contract, P_2 mean the contract is legal, P_3 mean Rogers has not performed the contract, and P_4 mean Rogers has accepted Jones' offer. Then,

$$((((((P_1 \wedge P_2) \wedge P_3) \implies P_0) \wedge (P_0 \implies ((P_1 \wedge P_2) \wedge P_3))) \wedge (P_1 \implies P_4)) \wedge \neg P_4) \implies \neg P_0$$

2.

(a) Define inputs $f : [P] \times [Q] \mapsto [R]$, $p : [P]$, and $q : [Q]$. Then, the evidence is

$$\lambda f. \, (\lambda p. \, (\lambda q. f \, (p, q)))$$

(b) Define inputs $f : [P] \mapsto (g : [Q] \mapsto [R])$ and $x : [P] \times [Q]$. Then, the evidence is

$$\lambda f. \, (\lambda x. f \, (x_1) \, (x_2))$$

(c) Define inputs $x : ([P] \times [Q] \mapsto \{\}, ([P] + (f : [P] \mapsto \{\}), ([Q] + (g : [Q] \mapsto \{\}), [P] \times [Q] \mapsto \{\})))$ and $y \in [P]$. One possible evidence is

$$\lambda x. case \, x_{21} \, of \, \text{inl} \, (p) \longrightarrow (case \, x_{221} \, of \, \text{inl} \, (q) \longrightarrow \text{inl} \, (\lambda y. x_1 \, (y, q)) \mid \text{inr} \, (g) \longrightarrow \text{inr} \, (g)) \mid \text{inr} \, (f) \longrightarrow \text{inl} \, (f)$$

3. Note: I did not list redundant hypotheses. The number after any left refinement rule indicates the position of the hypothesis acted upon in the hypothesis list.

(a) $(P \implies Q) \vee \neg (P \implies Q)$

$(P \vee \neg P), (Q \vee \neg Q) \vdash (P \implies Q) \vee \neg (P \implies Q)$ `by orL (1)`
  $P, (Q \vee \neg Q) \vdash (P \implies Q) \vee \neg (P \implies Q)$ `by orL (2)`
    $P, Q \vdash (P \implies Q) \vee \neg (P \implies Q)$ `by orR1`
      $P, Q \vdash (P \implies Q)$ `by impliesR`
        $P, Q \vdash Q$ `by axiom`
    $P, \neg Q \vdash (P \implies Q) \vee \neg (P \implies Q)$ `by orR2`
      $P, \neg Q \vdash \neg (P \implies Q)$ `by notR`
        $P, \neg Q, (P \implies Q) \vdash$ `f` `by impliesL (3)`
          $P, \neg Q, (P \implies Q) \vdash P$ `by axiom`
          $P, \neg Q, Q \implies$ `f` `by notL (2)`
            $P, \neg Q, Q \vdash Q$ `by axiom`
  $\neg P, (Q \vee \neg Q) \vdash (P \implies Q) \vee \neg (P \implies Q)$ `by orL (2)`
    $\neg P, Q \vdash (P \implies Q) \vee \neg (P \implies Q)$ `by orR1`
      $\neg P, Q \vdash (P \implies Q)$ `by impliesR`

$\neg P, Q, P \vdash Q$ `by axiom`
$\neg P, \neg Q \vdash (P \implies Q) \vee \neg(P \implies Q)$ `by orR1`
$\neg P, \neg Q \vdash (P \implies Q)$ `by impliesR`
$\neg P, \neg Q, P \vdash Q$ `by notL (1)`
$\neg P, \neg Q, P \vdash P$ `by axiom`

(b) $(P \wedge Q) \vee \neg(P \wedge Q)$

$(P \vee \neg P), (Q \vee \neg Q) \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orL (1)`
$P, (Q \vee \neg Q) \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orL (2)`
$P, Q \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orR1`
$P, Q \vdash (P \wedge Q)$ `by andR`
$P, Q \vdash P$ `by axiom`
$P, Q \vdash Q$ `by axiom`
$P, \neg Q \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orR2`
$P, \neg Q \vdash \neg(P \wedge Q)$ `by notR`
$P, \neg Q, (P \wedge Q) \vdash$ `f` `by andL (3)`
$P, \neg Q, Q \vdash$ `f` `by notL (2)`
$P, \neg Q, Q \vdash Q$ `by axiom`
$\neg P, (Q \vee \neg Q) \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orL (2)`
$\neg P, Q \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orR2`
$\neg P, Q \vdash \neg(P \wedge Q)$ `by notR`
$\neg P, Q, (P \wedge Q) \vdash$ `f` `by andL (3)`
$\neg P, Q, P \vdash$ `f` `by notL (1)`
$\neg P, Q, P \vdash P$ `by axiom`
$\neg P, \neg Q \vdash (P \wedge Q) \vee \neg(P \wedge Q)$ `by orR2`
$\neg P, \neg Q \vdash \neg(P \wedge Q)$ `by notR`
$\neg P, \neg Q, (P \wedge Q) \vdash$ `f` `by andL (3)`
$\neg P, \neg Q, P, Q \vdash$ `f` `by notL (1)`
$\neg P, \neg Q, P, Q \vdash P$ `by axiom`

(c) $(P \vee Q) \vee \neg(P \vee Q)$

$(P \vee \neg P), (Q \vee \neg Q) \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orL (1)`
$P, (Q \vee \neg Q) \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orL (2)`
$P, Q \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orR1`
$P, Q \vdash (P \vee Q)$ `by orR1`
$P, Q \vdash P$ `by axiom`
$P, \neg Q \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orR1`
$P, \neg Q \vdash (P \vee Q)$ `by orR1`
$P, \neg Q \vdash P$ `by axiom`
$\neg P, (Q \vee \neg Q) \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orL (2)`
$\neg P, Q \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orR1`
$\neg P, Q \vdash (P \vee Q)$ `by orR2`
$\neg P, Q \vdash Q$ `by axiom`
$\neg P, \neg Q \vdash (P \vee Q) \vee \neg(P \vee Q)$ `by orR2`
$\neg P, \neg Q \vdash \neg(P \vee Q)$ `by notR`
$\neg P, \neg Q, (P \vee Q) \vdash$ `f` `by orL (3)`
$\neg P, \neg Q, P \vdash$ `f` `by notL (1)`
$\neg P, \neg Q, P \vdash P$ `by axiom`
$\neg P, \neg Q, Q \vdash$ `f` `by notL (2)`
$\neg P, \neg Q, Q \vdash Q$ `by axiom`