CS 486: Applied Logic:

Handout on Blocked Tableau and Sequent Systems

Block tableau rules "spelled out" (based on Smullyan 20)

We consider sets S of formulas, and isolate the one we're interested in.

left	right
α S, T(A $\wedge$ B) S, T(A), T(B)	S, F(A $\wedge$ B) β S, F(A) S, F(B)
β S, T(A $\vee$ B) S, T(A) S, T(B)	S, F(A $\vee$ B) α S, F(A) , F(B)
β S, T(A $\supset$ B) S, F(A) S, T(B)	S, F(A $\supset$ B) α S, T(A), F(B)
α S, T($\sim$A) S, F(A)	S, F($\sim$A) α S, T(A)
* S, T(A), F(A)	

Gentzen Systems: multi-conclusioned sequent rules
(Smullyan 105/106)

Hypothesis and conclusion consist of sets of formulas (H, G).

left	right
$\wedge$L H, A $\wedge$ B $\vdash G$ H, A, B $\vdash G$	$H \vdash G$, A $\wedge$ B $\wedge$R $H \vdash G$, A $H \vdash G$, B
$\vee$L H, A $\vee$ B $\vdash G$ H, A $\vdash G$ H, B $\vdash G$	$H \vdash G$, A $\vee$ B $\vee$R $H \vdash G$, A, B
$\supset$L H, A $\supset$ B $\vdash G$ $H \vdash G$, A H, B $\vdash G$	$H \vdash G$, A $\supset$ B $\supset$R H, A $\vdash G$, B
$\sim$L H, $\sim$A $\vdash G$ $H \vdash G$, A	$H \vdash G$, $\sim$A $\sim$R H, A $\vdash G$
axiom H, A $\vdash G$, A	

Refinement Logic: Single-conclusioned sequent rules

Refinement Logic as implemented in NUPRL uses a slightly different notation for logical connectives. Instead of sets of formulas we consider lists (H, H'). Sequents only have a single formula G as conclusion.

We use a slightly different notation for logical connectives. Implication is now $\Rightarrow$ (instead of $\supset$), negation is $\neg$ (instead of $\sim$). Negation $\neg A$ is viewed as abbreviation for $A \Rightarrow \mathsf{f}$

left		right	
`andL`	$H,\ \mathrm{A} \wedge \mathrm{B},\ H' \vdash \mathrm{G}$ $H,\ \mathrm{A},\ \mathrm{B},\ H' \vdash \mathrm{G}$	$H \vdash \mathrm{A} \wedge \mathrm{B}$ $H \vdash \mathrm{A}$ $H \vdash \mathrm{B}$	`andR`

`orL` i	$H,\ \mathrm{A} \vee \mathrm{B},\ H' \vdash \mathrm{G}$
	$H,\ \mathrm{A},\ H' \vdash \mathrm{G}$
	$H,\ \mathrm{B},\ H' \vdash \mathrm{G}$
		$H \vdash \mathrm{A} \vee \mathrm{B}$
		$H \vdash \mathrm{A}$
		
		$H \vdash \mathrm{A} \vee \mathrm{B}$
		$H \vdash \mathrm{B}$
			`orR1`
			
			`orR2`

`impL` i	$H,\ \mathrm{A} \Rightarrow \mathrm{B},\ H' \vdash \mathrm{G}$ $H,\ \mathrm{A} \Rightarrow \mathrm{B},\ H' \vdash \mathrm{A}$ $H,\ H',\ \mathrm{B} \vdash \mathrm{G}$	$H \vdash \mathrm{A} \Rightarrow \mathrm{B}$ $H,\ \mathrm{A} \vdash \mathrm{B}$	`impR`
`notL` i	$H,\ \neg\mathrm{A},\ H' \vdash \mathrm{G}$ $H,\ \neg\mathrm{A},\ H' \vdash \mathrm{A}$	$H \vdash \neg\mathrm{A}$ $H,\ \mathrm{A} \vdash \mathsf{f}$	`notR`
`falseL` i	$H,\ \mathsf{f},\ H' \vdash \mathrm{G}$		
`axiom` i	$H,\ \mathrm{A},\ H' \vdash \mathrm{A}$		

Special Rules			
`magic` A	$H \vdash \mathrm{G}$ $H,\ A \vee \neg A \vdash \mathrm{G}$	$H \vdash \mathrm{G}$ $H \vdash A$ $H,\ A \vdash \mathrm{G}$	`cut` A

In the computerized version, all left rules must provide an index i of the hypothesis to indicate the formula to which the rule shall be applied. In `magic` and `cut` the formula A has to be provided.