Applied Logic	**Homework 8**
CS 4860 Spring 2009	**Due Date: Thursday, April 30, 2009**

Reading

The final three lectures will review the material that we have covered so far, elaborate some of the issues a bit deeper, and discuss the philosphical implications of the results and methods used.

Please prepare questions that you would like to see adressed in these lectures.

Questions

(1) Prove $(\forall x)(\exists y)(0 \neq x \supset y{+}1 = x)$ in Peano Arithmetic.

If you were to prove this formula in refinement logic, what well-known function would the evidence term describe?

(2) Show how to represent the following functions in Peano Arithmetic.

(a) Integer division *div* with $\mathit{div}(x, y) = x \div y$

(b) The function *divides* with $\mathit{divides}(x, y) = \begin{cases} 1 & \text{if } x \text{ divides } y \\ 0 & \text{otherwise} \end{cases}$

(c) The function *prime* with $\mathit{prime}(x) = \begin{cases} 1 & \text{if } x \text{ is a prime number} \\ 0 & \text{otherwise} \end{cases}$

(3) Show by providing an appropriate model that the following laws are not valid in $\mathcal{Q}$.

(a) $(\forall x, y)\ (x + y = y + x)$

(b) $(\forall x, y, z)\ (x + (y + z) = (x + y) + z)$

(c) $(\forall x)\ (0 + x = x)$

(d) $(\forall x, y)\ (x * y = y * x)$

(e) $(\forall x)\ (0 * x = 0)$

(4) **Bonus** Let $Prov$ be a provability predicate for the theory $\mathcal{Q}$ and X and Y be formulas in the language of $\mathcal{Q}$. Assume $\models_Q Prov(\lceil X \rceil) \supset Y$ and $\models_Q Prov(\lceil Y \rceil) \supset X$

Show, using Löb's theorem, that both X and Y are theorems in $\mathcal{Q}$.

Training Material

Here are some more questions that are similar to those that could be asked in an exam. You may use them to prepare yourself. They will not be graded.

(1) Give a Tableau proof of $(p \wedge (q \wedge (r \wedge ((p \vee r) \supset t)))) \supset t$.

(2) Prove the following formula in Refinement logic
$(\forall x)[(\exists y)P(x,y) \supset P(x,x)] \supset (\forall z)(\forall t)[P(z,t) \supset (\exists y)P(x,y)]$

(3) In the Tableau rules, there is a proviso on the δ rules.

  (a) If the proviso is removed, is the proof system *consistent*? Explain your answer as cogently as possible, using ideas from the course. The quality of your explanation is very important.

  (b) If the proviso is removed, is the proof system *complete*? Explain your answer.

(4) Consider the boolean ring $\langle \mathbb{B}$, =, $\Leftrightarrow$, $\vee$,$\rangle$ with the identities , F. Define the operations $\sim$, $\wedge$, and $\supset$ in terms of the ring operations and prove the followng laws solely on the basis of the ring axioms.

(1) $p \supset (p \vee q)$,
(2) $(p \wedge q) \supset p$,
(3) $(p \wedge q) \supset q$,
(4) $p \supset (q \supset p)$,
(5) $\sim q \supset (q \supset p)$,
(6) $p \supset q \supset (\sim q \supset \sim p)$, and
(7) $p \vee p \Leftrightarrow p$.