Applied Logic | **Homework 7**
CS 4860 Spring 2009 | **Due Date: Tuesday, April 21, 2009**

Reading

Please read the notes on Arithmetic Representability (# 22) for Tuesday, April 14.

Please read the notes on Unsolvable Problems in Logic (# 23) for Thursday, April 16.

Please read the notes on the theory $\mathcal{Q}$ and the undecidability of logic (# 24) for Tuesday, April 21.

Questions

(1) Prove the law of transitivity for equality using only the rules of refinment logic and an appropriate instance of the substitution axiom scheme.

subst instance ⊢ `(∀x,y,z) ((E(x,y) ∧ E(y,z)) ⊃ E(x,z))`

(2) a) Give a finite model of a semigroup that is not commutative.

b) Give a finite model of a commutative semigroup that is not a monoid.

(3) Let n be an arbitrary natural number. Under which conditions is $\mathbb{Z}_n \equiv \langle \mathbb{Z}, =_n, +, * \rangle$ is a field?

Explain why the field axioms are satisfied if $\mathbb{Z}_n$ is a field and which axiom is violated if $\mathbb{Z}_n$ is not a field.

(4) Define `x<y ≡ (∃z)(x+z+1 = y)` and prove the seven axioms of discrete linear orders for `<` from the Peano axioms.

```
lt-asym:     (∀x,y) (x<y ⊃ ∼(y<x))
lt-trans:    (∀x,y,z) ((x<y ∧ y<z) ⊃ x<z)
lt-linear:   (∀x,y) (x<y ∨ y<x ∨ x=y)
lt-discrete: (∀x,y) ∼(x<y ∧ y<x+1)
lt-0-1:      0<1
lt-mono-+:   (∀x,y,z)(x<y ⊃ x+z < y+z)
lt-mono-*:   (∀x,y,z)((0<z ∧ x<y) ⊃ x*z < y*z)
```

Submit your proofs for `lt-0-1` and `lt-discrete`.