Reading

Please read pp. 43–57 in Smullyan for Thursday, March 12
and pp. 57–65 in Smullyan for Tuesday, March 24.

Project Work

During the second half of this course you should work on a self-chosen project related to the topic of applied logic. This could, for instance, be a literature study about an interesting or the implementation (and documentation) of a proof environment. We will discuss a few possibilities in class. Please prepare a project proposal (about half a page) for Tuesday, March 31.

Questions

1. Prove the following formulas in refinement logic
   (a) $(P \Rightarrow Q) \Rightarrow \neg Q \Rightarrow \neg P$
   (b) $\neg(P \wedge Q) \Rightarrow \neg P \vee \neg Q$
   (c) $\neg\neg P \Rightarrow P$

2. Prove that the `magic` rule is equivalent to the following rule

   $H \vdash G$ by `contradiction`
   $\quad H, \neg G \vdash \mathtt{f}$

   Show how to derive `contradiction` in refinement logic and how to derive `magic` in a refinement logic with `contradiction` (and without `magic`).

   Note that using the `cut` *rule is not necessary but simplifies the derivation.*

3. For the completeness proof of refinement logic we have introduced the following rules.

   $H \vdash A \vee (B \vee C)$ by `orAssocL`
   $\quad H \vdash (A \vee B) \vee C$

   $H \vdash A \Rightarrow B \vee G^*$ by `impR*`
   $\quad H, A \vdash B \vee G^*$

   Prove that both can be *derived* in refinement logic by giving proof fragments that simulates them.

4. Prove or disprove these P^2 formulas:
   (a) $(\forall p)(\forall q)\ ((p \Rightarrow q) \Rightarrow ((p \Rightarrow \bot) \Rightarrow (q \Rightarrow \bot)))$
   (b) $(\forall p)\neg p \Rightarrow \neg((\exists p)p)$
   (c) $(\forall p)(\exists q)\ ((p \vee q) \Rightarrow p)$
   (d) $(\forall A)(\forall B)\ (A \vee B \Rightarrow (\forall p)((A \Rightarrow p) \Rightarrow (B \Rightarrow p) \Rightarrow p))$