CS 486: Spring 2001
Cornell University
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Feb 20 Lecture Notes: Hintikka

$\forall S : \mathbb{B} \times Form \rightarrow \mathbb{B}.\ Hintikka(S) \Rightarrow \exists v : Var \rightarrow \mathbb{B}.\ \forall X : Form.(S(X) \Rightarrow \mathtt{s}value(X, v) = t)$

Proof: Define v as follows for $p \in Var$

$v(p) = S(< T, p >)$ i.e. assign t top if Tp is in S.

(this assigns f if $< F, p > \in S$ and if neither $< T, p >$ nor $< F, p >$ is in S).

Claim $\forall X : Form.(S(X) \Rightarrow \mathtt{s}value(X, v) = t)$

Proof: by induction on $Form$.

base: X is a signed variable;

if $< T, p > \in S$ then $\mathtt{s}value(X, v) = t$, since $v(p) = t$.

if $< F, p > \in S$ then $\mathtt{s}value(X, v) = t$, since $v(p) = f$.

induction: assume the result for all U, V subformulas of X

Case X is an α formula: then α_1, α_2 are both subformulas of α, and since S is Hintikka, $S(\alpha_1)$ and $S(\alpha_2)$. So by induction, $\mathtt{s}value(\alpha_1, v) = t$ and $\mathtt{s}value(\alpha_2, v) = t$, thus $\mathtt{s}value(\alpha, v) = t$.

X is a β formula: then β_1, β_2 are both subformulas of β, and since S is Hintikka, one of them is in S, say $S(\beta_i)$. Thus by induction $\mathtt{s}value(\beta_i, v) = t$, hence $\mathtt{s}value(\beta, v) = t$.

Qed