CS 486: Spring 2001
Cornell University
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Feb 15 Lecture Notes: Consistency

Theorem $\forall X : Form.\ \forall t : \text{Tableau}_X.\ \forall v_0 : var \to \mathbb{B}.$
$(\mathtt{s}value(root(t), v_0) = t) \Rightarrow$
$\exists \Theta : Path(t).\ \forall Y : Form.(Y \text{ on } \Theta \to \mathtt{s}value(Y, v_0) = t)$

Proof: by induction on the depth of t.

base: t is a leaf, the root. Then the root is the path.

induction: suppose there is a path Θ in any tableau t_n of depth n
consider now a tree of depth $n+1$.
If Θ is a path to a leaf in t_{n+1}, then we are done.
If Θ is extended in t_{n+1}, then it is either
(A) (Θ, α_1) or (Θ, α_2) or
(B) (Θ, β_1) or (Θ, β_2)

In case (A), α is true since it is on Θ and so are $(\Theta, \alpha), (\theta, \alpha)$.
In case (B), β is true since it is on Θ and so on of (Θ, β_1), (Θ, β_2) is.
Pick the true one as the new path.

Qed

Lemma 1 $\forall X : Form.\ \forall t : \text{Tableau}_X.\ closed(t) \Rightarrow$
$\neg\exists\Theta : Path(t).\ \forall X : Form.(Y \text{ on } \Theta \Rightarrow \mathtt{s}value(Y, v_0) = t)$

Pf: If t is closed, then on every path Θ there appears TY and FY for some Y. So not all are true.

Qed

Corollary. $\forall X : Form.\ \forall t : \text{Tableau}_X.\ closed(t) \Rightarrow$
$\forall v_0 : Var_X \to \mathbb{B}.\ (\mathtt{s}value(FX, v_0 = f).$

Theorem: $\forall X : Form.\ (\exists t : \text{Tab closed}(t) \Rightarrow \text{tautology}(X))$