CS486 Applied Logic Assignment 7 Due Thursday, April 5, 2001

Reading: Please study all of Smullyan Chapter 5.

Exercises:

1. Prove by Refinement Logic
   a. $\forall x P(x) \vee \exists x. \neg P(x)$
   b. $(\exists y \forall x A(x,y) \Rightarrow \exists v \forall u B(u,v)) \Rightarrow$
      $\forall y \exists v \forall u \exists x (A(x,y) \Rightarrow B(u,v))$

2. Suppose that $\mathbb{N}_n = \{0,1,2,\ldots,n\text{-}1\}$. Show that if P(x) is a computable boolean expression, then there is a program to decide whether

   $$\forall x{:}\mathbb{N}_n . P(x) \quad \text{or} \quad \exists x{:}\mathbb{N}_n . \neg P(x).$$

   What can you say about the relationship of this program to the proof in problem 1?

3. Solve the exercise on p. 63 of Smullyan.