4 Mar 2022 — Continuous distributions and Gaussians

(Reading: §12.4)

<u>Announcements:</u> Quiz 1 grades to be released tonight.
On Gradescope / CMS (total grade)
(Make-up quiz yet to be graded.)

Without "drop lowest score" correction, here's how things look...

24 = perfect score

17+ $\longrightarrow$ A +/−

13–17 $\longrightarrow$ B +/−

10–13 $\longrightarrow$ C +/−

<10 $\longrightarrow$ you're probably omitting this quiz score.

We say random variables $X, X' \in \mathbb{R}^d$ have the same distribution iff

$$\forall_{\text{measurable}} \text{ set } S \subset \mathbb{R}^d \quad \Pr(X \in S) = \Pr(X' \in S).$$

To say that random variable $X \in \mathbb{R}^d$ has probability density f means

$$\forall S \quad \Pr(X \in S) = \int_S f(\vec{y})\, d\vec{y}$$

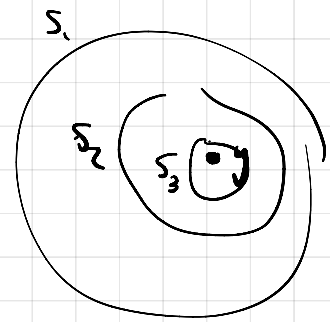
Density always satisfies $\int_{\mathbb{R}^d} f(y)\, dy = 1$, $\forall y \; f(y) \geq 0$.

FACT: If X, X' have same density, they have same distribution.

Obs. Not every distrib has a density,
e.g.
$X = \vec{0}$ wp probability 1.

When X has a density, the density $f(y)$ can be defined as

$$f(y) = \lim_{\substack{\mathrm{diam}(S) \to 0 \\ y \in S}} \frac{\Pr(X \in S)}{\mathrm{Vol}_d(S)},$$



For random variable $X \in \mathbb{R}^1$, the cumulative distribution function (CDF) is F defined by

$$F(\theta) = \Pr(X \in (-\infty, \theta])$$
$$= \Pr(X \leq \theta)$$

Useful fact: If X has a continuous CDF, F, then the random variable $F(X)$ is uniformly distributed in $[0,1]$.

Distribution of $Y = F(X)$, satisfies $\Pr(Y \in S) = \Pr(F(X) \in S)$ $\forall S$.

"Z is unif on $[0,1]$" means
$\forall\ 0 \leq a \leq b \leq 1$, $\quad \Pr(Z \in [a,b]) = b - a$.

If F is the (continuous) CDF of X and $Y = F(X)$, then why is Y un

If F is continuous and $F(X)$ is unif $[0,1]$ then F is the CDF of X.

$\Rightarrow$ if Y is Unif $[0,1]$ and F is continuous, strictly increasing then $X = F^{-1}(Y)$ has CDF F.

<u>Ex.</u> Exponential distribution with rate r is characterized by

$$\Pr(X > \theta) = e^{-r\theta}$$

$$F(\theta) = 1 - e^{-r\theta}$$

$F^{-1}(y) =$ the θ that solves

$$1 - e^{-r\theta} = y$$

$$\theta = \frac{1}{r} \ln\left(\frac{1}{1-y}\right).$$