$\boxed{\text{23 Feb 2022}}$

SVD interpretation

Pos Def matrices

Quiz review

$$
\begin{array}{c|cccc}
 & \multicolumn{4}{c}{\overbrace{\hspace{6cm}}^{\text{31 stats}}} \\
 & \text{PTS} & \text{WS} & \text{BLK} & \text{TRB} \cdots \\
\hline
\text{Player 1} & & & & \\
\text{Player 2} & & & & \\
\text{Player 3} & & & A & \\
\vdots & & & & \\
\text{Player 127} & & & &
\end{array}
$$

$$
D = \begin{bmatrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \ddots & \\ 0 & & & \sigma_{31} \\ \hline & & 0 & \end{bmatrix}
$$

$$
\underbrace{A}_{\text{Player} \times \text{Stat}} = \underbrace{U}_{\text{Player} \times \text{SVSEQ\#}} \underbrace{D}_{\text{SVSEQ\#} \times \text{PNZSVSEQ}} \underbrace{V^T}_{\text{PNZSVSEQ} \times \text{Stat}}
$$

$$
\underbrace{\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \sigma_4 \geq \cdots \geq \sigma_{31} \geq 0 \cdots \geq 0}_{\text{singular value seq.}}
$$

SVSEQ

PNZSVSQ "potentially nonzero"   ARZSVSEQ

A row of V^T (col. of V) is a vector in $(\mathbb{R}^{\text{Stat}})^*$: a coefficient vector to combine stats by weighted summation to produce a "meta-statistic" that is better aligned with the data than any single stat.

A row of $\mathcal{U}$ is one player's list of "meta-statistics".

A column of $\mathcal{U}$ focuses on some meta-stat and evaluates all players using that meta-stat.

Quiz review. List of topics is on Ed.

1. Vect Spc Fundamentals

2. Inner products + Duals

3. Convexity

4. Norms

5. Differentials + Gradients

6. Volumes in $\mathbb{R}^d$

7. Matrices.

E.g. The inner product $\left\langle \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \right\rangle = x_1 y_1 + \frac{1}{2} x_1 y_2 + \frac{1}{2} x_2 y_1 + x_2 y_2$

is non-degenerate.

$\begin{bmatrix}3\\-4\end{bmatrix} \mapsto \lambda y.\ \langle \begin{bmatrix}3\\-4\end{bmatrix}, \begin{bmatrix}y_1\\y_2\end{bmatrix} \rangle$

So it defines an isomorphism $\mathbb{R}^2 \to (\mathbb{R}^2)^*$.
Write the row vector in $(\mathbb{R}^2)^*$ corresponding to $\begin{bmatrix}3\\-4\end{bmatrix}$ under this iso.

$3y_1 + \frac{3}{2}y_2 - 2y_1 - 4y_2 = y_1 - \frac{5}{2}y_2$ $\quad\quad \begin{bmatrix}1 & -\frac{5}{2}\end{bmatrix}$.

E.g. For each of these functions, is it convex?

$$f(x,y) = x^2 - y^2$$
$$g(x,y) = x^2 - y$$
$$h(x,y) = (x-y)^2$$

E.g. Which of the following is true?

$$\lim_{d\to\infty} d^{\frac{1}{2}} \cdot \left(\mathrm{vol}_d\left(B_2^d(1)\right)\right)^{\frac{1}{d}} \text{ is finite}$$

$$\lim_{d\to\infty} d^{-1/2}\left(\mathrm{vol}_d(B_2^d(1))\right)^{\frac{1}{d}} \text{ is } > 0$$

both

neither

Fact. $\mathrm{vol}_d\left(B_2^d(1)\right) \approx \left(\frac{c}{\sqrt{d}}\right)^d$

$$\mathrm{vol}_d(B_2^d(1))^{\frac{1}{d}} \sim \frac{c}{\sqrt{d}}$$