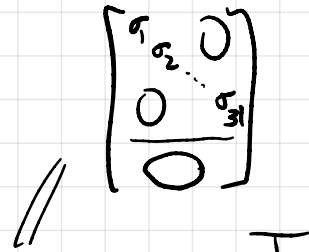
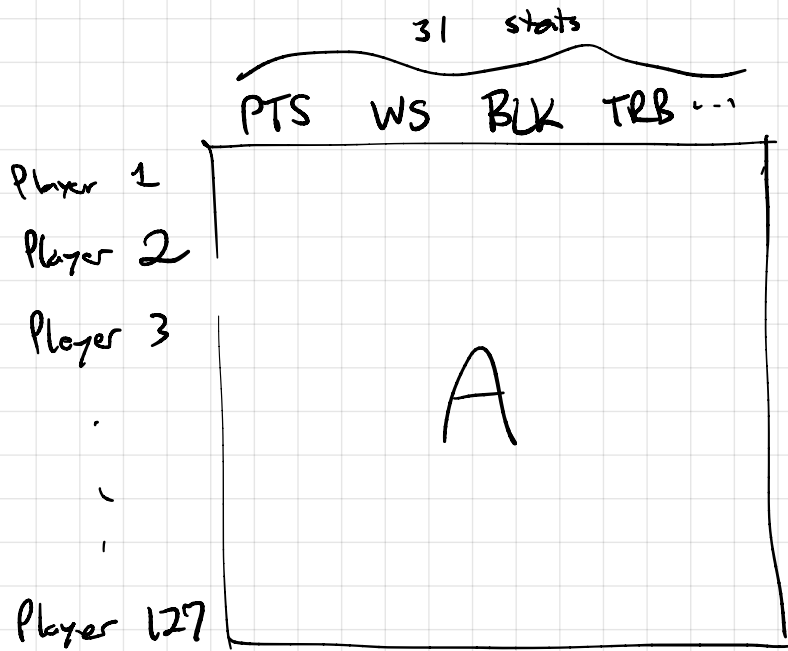


23 Feb 2022

SVD interpretation
 Pos Def matrices
 Quiz review



$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \sigma_4 \geq \dots \geq \sigma_{31} \geq 0 \dots \geq 0$$

singular value seq.

SVSEQ

PNZSVSEQ
 "potentially nonzero"

ARZSVSEQ

$$A = U D V^T$$

$\underbrace{\hspace{2cm}}_{\text{Player} \times \text{Stat}}$
 $\underbrace{\hspace{2cm}}_{\text{Player} \times \text{SVSEQ}}$
 $\underbrace{\hspace{2cm}}_{\text{SVSEQ} \times \text{PNZSVSEQ}}$
 $\underbrace{\hspace{2cm}}_{\text{PNZSVSEQ} \times \text{Stat}}$

A row of V^T (col. of V) is a vector in $(\mathbb{R}^{\text{Stat}})^*$: a coefficient vector to combine stats by weighted summation to produce a "meta-statistic" that is better aligned with the data than any single stat.

A row of U is one player's list of "meta-statistics".

A column of U focuses on one meta-stat and evaluates all players using that meta-stat.

Quiz review. List of topics is on Ed.

1. Vect Spc Fundamentals
2. Inner products + Duals
3. Convexity
4. Norms
5. Differentials + Gradients
6. Volumes in \mathbb{R}^d
7. Matrices.

E.g. The inner product $\left\langle \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \right\rangle = x_1 y_1 + \frac{1}{2} x_1 y_2 + \frac{1}{2} x_2 y_1 + x_2 y_2$ is non-degenerate.

So it defines an isomorphism $\mathbb{R}^2 \rightarrow (\mathbb{R}^2)^*$. $[-3] \mapsto \lambda y \langle [3], [1/2] \rangle$

Write the row vector in $(\mathbb{R}^2)^*$ corresponding to $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$ under this iso.

$$3y_1 + \frac{3}{2}y_2 - 2y_1 - 4y_2 = y_1 - \frac{5}{2}y_2 \quad \left[1 \quad -\frac{5}{2}\right]$$

E.g. For each of these functions, is it convex?

$$f(x,y) = x^2 - y^2$$

$$g(x,y) = x^2 - y$$

$$h(x,y) = (x-y)^2$$

E.g. Which of the following is true?

$$\lim_{d \rightarrow \infty} d^{\frac{1}{2}} \cdot \left(\text{vol}_d(B_2^d(1)) \right)^{\frac{1}{d}} \text{ is finite}$$

$$\lim_{d \rightarrow \infty} d^{-\frac{1}{2}} \left(\text{vol}_d(B_2^d(1)) \right)^{\frac{1}{d}} \text{ is } > 0$$

both

neither

Fact. $\text{vol}_d(B_2^d(1)) \approx \left(\frac{c}{\sqrt{d}} \right)^d$

$$\text{vol}_d(B_2^d(1))^{\frac{1}{d}} \sim \frac{c}{\sqrt{d}}$$