homework question

sphere, pick arbitrary point, make n north pole and get \( n \) to \( n \) distance

- pick 10 north poles, pick set of points
- show that all points are \( \pm 1 \) to all north poles

for homework, increase number of sphere

Phase transitions

\( G(n,p) \)

- store as property is monotone, there's a phase transition for
- \( p \) such that less than \( p \) doesn't have property and greater than \( p \) does have property
- when does graph have diameter 2 \( P = \sqrt{\frac{\log n}{n}} \)
- \( \frac{1}{2} < \frac{p}{\log n} \) such that \( \lim_{n \to \infty} \mathbb{P}(R(0) = 0) = 0 \)
- then \( G(n,p,\log n) \) doesn't have property
- \( \lim_{n \to \infty} \frac{2n}{\log n} = \infty \) then \( G(n,p,\log n) \) has property
- \( p(\log n) \) is threshold and there's phase transition

- often we have a random variable \( X \) that indicates how many copies of some item \( G(n,p) \) has.
- if \( X \) cycles, \( \lim_{n \to \infty} E(X) = 0 \) \( \Rightarrow \) \( \text{prob(graph has item)} = 0 \)

<table>
<thead>
<tr>
<th>( n ) graph</th>
<th>( n ) cycles</th>
<th>( 0 % )</th>
<th>( 10 % )</th>
<th>( 20 % )</th>
<th>( 40 % )</th>
<th>( 60 % )</th>
<th>( 80 % )</th>
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<tbody>
<tr>
<td>( E(X) = 1 )</td>
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Second moment method is used to show that when the expected value of a non-negative random variable is large compared to its variance then (random variable takes on value \( 0 \) with probability 0)

\( \text{prob}(X = 0) \leq \text{prob}(1 \cdot (X - E(X)) = E(X)) \)

by chernoff's inequality \( (\log (2X - E(X)) \leq \frac{X}{2}) \)

\( \text{prob}(X = 0) \leq \text{prob}(1 \cdot (X - E(X)) = E(X)) \leq \frac{e^{-\frac{1}{2}}}{E(X)} \)

\( E(X) = \alpha - \frac{\alpha}{\epsilon^2} \)

can claim \( \alpha \) \( \Rightarrow \) \( \text{prob(graph has item)} > 0 \) if \( \lim \frac{\epsilon^2}{\alpha} = 0 \)
\[ E(x) = 0 \]

\[ E(x) = \begin{cases} \infty & \text{if } n/\sqrt{\log n} \to 0 \\ 0 & \text{otherwise} \end{cases} \]

\[ \lim_{n \to \infty} \frac{E(x)}{\sqrt{\log n}} = 0 \]

**Proof:** If \( n \) has diameter \( d \), then \( d \) vertices \( w \) and \( v \) are adjacent. Since \( w \) and \( v \) distinguish it from \( \emptyset \),

\[ \begin{align*}
E(x) &= \left( \sum_{i \leq n/\sqrt{\log n}} \right) \left( \sum_{j \leq n/\sqrt{\log n}} \right) \\
&= \sum_{i \leq n/\sqrt{\log n}} \sum_{j \leq n/\sqrt{\log n}} \\
&= n^2 \left( \frac{1}{\log n} \right)^2 \\
&= \frac{n^2}{\log^2 n} \\
&\to 0 \quad \text{as } n \to \infty
\end{align*} \]

Threshold for diameter \( d \): \[ p = \sqrt{\frac{n}{\log n}} \]

For \( c < \sqrt{\frac{1}{\log n}} \), \( G(n, p) \) almost surely has diameter less than or equal to 2.