Homework assignment 6 is due Friday March 12

1. (a) What is the set of possible harmonic functions on a graph if there are only interior vertices and no external vertices that supply the boundary condition.

(b) Let q_x be the stationary probability of vertex x in a random walk on an undirected graph and let d_x be the degree of vertex x. Show that $\frac{q_x}{d_x}$ is a harmonic function.

(c) If there are multiple harmonic functions when there are no boundary conditions why is the stationary probability of a random walk on an undirected graph unique?

(d) What is the stationary probability of a random walk on an undirected graph.

2. Given a graph consisting of a single path of five vertices numbered 1 to 5, what is the probability of reaching vertex 1 before vertex 5 when starting at vertex 4.

3. (Optional) Prove that reducing the value of a resistor in a network cannot increase the effective resistance. Try to prove it first for series-parallel networks.

4. Prove that the escape probability $p_{escape} = \frac{c_{eff}}{c_a}$ must be less than one.

5. What is the hitting time h_{uv} for two adjacent vertices on a cycle of length n? What is the hitting time if the edge u,v is removed?

   (Some hints on solving non-homogeneous second order difference equations: If you want to solve a difference equation of the form:

   c1 h_{i-1}+c2 h_i + c3 h_{i+1} = d then the solution is the superposition of the solution to the homogeneous equation plus a particular solution to the non-homogeneous.

   h_i = h_i^{hom}+h_i^{part}

   Where h_i^{hom} is the solution to c1 h_{i-1}+c2 h_i + c3 h_{i+1} = 0 and h_i^{part} is any function of i that satisfies c1 h_{i-1}+c2 h_i + c3 h_{i+1} = d (Hint: for the difference equation that might arise in your problem try using $h_i^{part}=i^2$). The way of solving homogeneous difference equations was described in class and can also be found in last years notes:

   http://www.cs.cornell.edu/courses/cs4850/2009sp/Scribe%20Notes/Lecture%2023%20Wednesday%20March11.pdf

   After you find a general solution h_i you have to calculate the constants that remain in your solution by taking border cases of your recursion, e.g. what happens at h_1 or h_0 or h_n etc.)

6. Consider the set of integers $\{1, 2, \cdots, n\}$. How many draws d with replacement are necessary so that every integer is drawn?

This figure is for the next homework: