Homework assignment 5 is due Friday March 5

1. Let *A* be a square $n \times n$ matrix whose rows are orthonormal. Prove that the columns of *A* are orthonormal.

2. Suppose *A* is a $m \times d$ matrix with block diagonal structure where the blocks $B_1, B_2, \cdots, B_k$ are $\frac{m}{k} \times \frac{d}{k}$ and all entries of each B_i are a_i with $a_1 > a_2 > \cdots > a_k > 0$. Show that *A* has exactly *k* nonzero singular vectors $\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_k}$ where v_i has the value $(\frac{k}{d})^{1/2}$ in coordinates $(i-1)\frac{d}{k}+1, (i-1)\frac{d}{k}+2, \cdots, i\frac{d}{k}$ and 0 elsewhere. In other words, the singular vectors exactly identify the blocks of the diagonal.

3. Interpret the right and left singular vectors for the document term matrix.

4. Verify that the sum of rank one matrices $\sum_{i=1}^{r} \sigma_i \mathbf{u_i} \mathbf{v_i}^T$ can be written as UDV^T, where the $\mathbf{u_i}$ are the columns of U and $\mathbf{v_i}$ are the columns of V. To do this, first verify that for any two matrices P and Q, we have

$$PQ = \sum_i \mathbf{p_i} \mathbf{q_i}$$

where, $\mathbf{p_i}$ is the i th column of P and $\mathbf{q_i}$ is the i th row of Q.

You have an option to do either number 5 or number 6 which ever you prefer

5. Read in a photo and convert to a matrix. Perform a singular value decomposition. Reconstruct the photo using only 10%, 25%, 50% of the singular values. (a) Print the reconstructed photo. How good is the quality of the reconstructed photo?
(b) What percent of the Forbenius norm is captured in each case?
Hint: If you use Matlab the command to read a photo is imread. The types of files that can be read are given by imformats, to print the file use imwrite. Print using jpeg format. To access the file afterwards you may need to add the file extension .jpg. imread will read the file in uint8 and you will need to convert to double for the svd code. Afterwards you will need to convert back to uint8 to write the file. If the photo is a color photo you will get three matrices for the three colors used.

6. (a) Generate a random 1000 by 1000 matrix and find the singular values. Plot the singular values. By plot we mean sort the values and draw the curve where the x-axis is the integers one to 1000 and the y axis is the value of the i^{th} singular value. Create a histogram of the singular values. You may want to ignore the first singular value if it is much larger than the remainder.
(b) Generate a random 1000 by 1000 matrix made up of 100 by 100 blocks. Multiply the elements in the diagonal blocks by some constants greater than one and multiply the elements in the off diagonal block by 0.1. Find and plot the singular values.