Homework assignment 3 due Friday February 19
To expedite grading please submit each problem on a separate sheet of paper.

1. Prove that $1+x \le e^x$ for all real *x*. For what values of *x* is the approximation $1+x = e^x$ good?

2. Let $f(n)$ be a function that is asymptotically less than *n*. Some such functions are *1/n*, a constant *d*, $\log n$ or $n^{\frac{1}{3}}$. Show that

$$\lim_{n\to\infty}\left(1+\tfrac{f(n)}{n}\right)^n = e^{f(n)}.$$

3. Let $G(n,p)$ be a random graph and let *x* be the random variable denoting the number of unordered pairs of non adjacent vertices *(u,v)* such that no other vertex of *G* is adjacent to both *u* and *v*. Prove that if $\lim_{n\to\infty} E(x) = 0$, then for large *n* there are almost no disconnected graphs i.e. $\text{Prob}(x=0) \to 1$ and hence $\text{Prob}(\text{G is connected}) \to 1$. Actually the graph becomes connected long before this condition is true.

4. Let x_i, $1 \le i \le n$, be a set of indicator variables with identical probability distributions. Let $x = \sum_{i=1}^{n} x_i$ and suppose $E(x) = \infty$. Show that if the x_i are statistically independent, then $\text{Prob}(x=0) = 0$.

5. In the proof that every monotone property has a threshold we cannot say that $G(n,q)$ has the property Q only if one of the $G(n, p(\varepsilon))$ has the property Q even though $G(n,q)$ is the union of the $G(n, p(\varepsilon))$. $G(n,q)$ might have the property even though none of the $G(n, p(\varepsilon))$ have the property. Give an example of such a property.

6. Consider a model of a random subset $N(n,p)$ of integers $\{1,2,\ldots n\}$ where, $N(n,p)$ is the set obtained by independently at random including each of $\{1,2,\ldots n\}$ into the set with probability p. What is the threshold for $N(n,p)$ to contain a) a perfect square, b) a perfect cube, c) an even number, d) three numbers such that x+y=z.