Homework Assignment 12 is due Wednesday May 5

1. Let p be a prime. A set of hash functions $H = \{h \mid \{0,1,\cdots, p-1\} \to \{0,1,\cdots, p-1\}\}$ is 3-universal if for all u,v,w,x,y, and z in $\{0,1,\cdots, p-1\}$ the $\text{Prob}(h(x)=u, h(y)=v, h(z)=w)$ equals $\frac{1}{p^3}$. Is the set of hash functions $H = \{h(x) = ax+b \bmod p \mid a,b \in \{0,1,2,\cdots, p-1\}\}$ 3-univresal?

2. Give an example of a set of hash functions that is not 2-universal.

3. Assume $x_i \in \{0,1\}$, $1 \le i \le 6$. Let $p(x_1, x_2, \cdots, x_6) = \frac{1}{Z}(x_1 + x_2 + x_3)(x_1 + x_4 + x_5)(x_2 + x_6)$. Execute the message passing algorithm discussed in class to calculate the six marginal probabilities $p(x_i) = \sum p(x_1, x_2, \cdots, x_6)$, $1 \le i \le 6$.