Homework 8 due March 27

Select five of the following problems. Feel free to substitute a problem of your own making.

1. Consider a random walk in an undirected graph. Assume at each vertex there is a probability distribution telling you the probability that you should select a given edge. That is, the edges at a given vertex are not equally likely to be chosen. What can you say about the frequency of traversing a given edge in both directions? Are the two frequencies always the same?

2. Given a graph consisting of a single path of five vertices numbered 1 to 5, what is the probability of reaching vertex 1 before vertex 5 when starting at vertex 4.

3: Prove that reducing the value of a resistor in a network cannot increase the effective resistance.

4: Prove that the escape probability $p_{escape} = \frac{c_{eff}}{c_a}$ must be less than one.

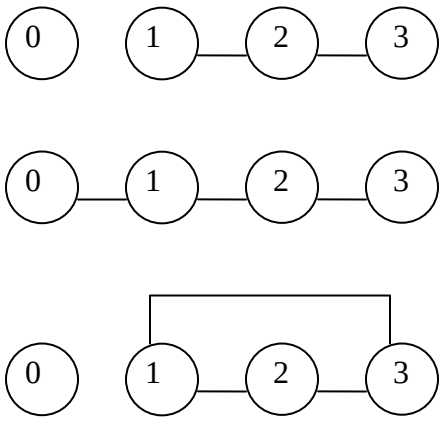
5. What is probability of returning to start vertex on a random walk on a planar graph?

6: Create a model for a graph similar to a three dimensional lattice in the way that a planar graph is similar to a two dimensional lattice. What is probability of returning to the start vertex in your model?

7. What is the hitting time h_{uv} for two adjacent vertices on a cycle of length n? What is the hitting time if the edge u,v is removed?

8. Consider the set of integers $\{1, 2, \cdots, n\}$. How many draws d with replacement are necessary so that almost surely every integer is drawn?

9. Show that adding an edge can either increase or decrease hitting time by calculating h_{13} for the following three graphs.



10. Prove that two independent random walks on a two dimensional lattice will hit with probability one.

11. Determine by simulation the escape probability for the 3-dimensional lattice.

12. What is the escape probability for a random walk starting at the root of a binary tree?

13. Consider a random walk on the positive half line, that is the integers 0,1,2,…
At the origin always move right one step. At all other integers move right with probability 2/3 and left with probability 1/3. What is the escape probability?

14. Consider a random walk from vertex a to vertex b in an undirected graph. Is it true that the expected number of net traversals of edge *xy* is less than or equal to one? Why?