

Do the project and select five problems from the list below that interest you and solve them.

**Project:** Pick a research area that interests you and do a literature search. List the three most important papers and give a two or three sentence explanation about why each paper or book is important. If you were going to automate this process how would you do it?

### Branching processes

**0.** Let  $p_0, p_1, p_2, \dots$  be the probability of  $i$  children. Let  $z_1$  be the number of children in the first generation. What is the expected value of  $z_1$ ? If it is the slope of  $f(x)$  at  $x=1$  explain intuitively why this is so.

### Generating functions

**1:** A dice has six faces, each face of the dice having one of the numbers 1 through 6. The result of a role of the dice is the integer on the top face. Consider two roles of the dice. For each integer 1,2,3... in how many ways can it be the sum of two roles of the dice.

**2:** If  $a(x)$  is the generating function for the sequence  $a_0, a_1, a_2, \dots$ , for what sequence is  $a(x)(1-x)$  the generating function.

**3:** How many ways can one draw  $n$  a's and b's with an even number of a's.

**4:** Find the generating function for the recurrence  $a_i = 2a_{i-1} + i$  where  $a_0 = 1$ .

**5:** Find a closed form for the generating function for the infinite sequence of perfect squares 1, 4, 9, 16, 25, ...

**6:** Given that  $\frac{1}{1-x}$  is the generating function for the sequence 1,1,..., for what sequence is  $\frac{1}{1-2x}$  the generating function?

**7:** The generating function for the Catalan numbers is  $c(x) = \frac{1 - \sqrt{1-4x}}{2x}$ . Find an approximation for  $c_n$  by expanding  $\sqrt{1-4x}$  by the binomial expression and using Sterling's approximation for  $n!$ .

### Growth models

**8.** Do a careful computer simulation to determine what happens when  $\delta = 1/8$  in the simple growth model without preferential attachment.