Homework 4 due Feb 20

Either (1) put some effort into the research project or (2) write the essay and do one of the problems.

Research project: $G\left(n,\frac{1}{2}\right)$ has a clique of size $\left(2-\varepsilon\right)\log n$ but there is no polynomial time algorithm to find it. There is a simple algorithm to find a clique of size logn. The problem is related to P and NP. Juels and Peinado, SODA 1998.
There seems to be a similar issue with finding a satisfying assignment for a formula in CNF. For small number of clauses we can easily find a satisfying assignment. For more clauses there is a region where the formula is sure satisfiable but we cannot find a satisfying solution. Is there any relationship between these two problems?
Locate the literature on these two problems and write a short summary of what is known.

Essay: Write a short essay that proves that every monotonic property of G(n,p) has a threshold. The essay should be written to stress the intuitive reasoning why the theorem is true. Try for something that could be understood by a bright high school senior.

Problem: Consider generating points in d-dimensions according to a unit variant Gaussians. The probability distribution for x_i is $\frac{1}{\sqrt{2\pi}}e^{-\frac{x_i^2}{2}}$. The probability distribution for x is $\frac{1}{(2\pi)^{\frac{d}{2}}}e^{-\frac{x_1^2+x_2^2+\cdots+x_d^2}{2}}$. To show that the distribution is spherically symmetric I would write the probability distribution as a function of r as $\frac{1}{\sqrt{2\pi}}e^{-\frac{r^2}{2}}$. But something seems wrong since the angle is $A_d=\frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}$. What is wrong?

Problem: Return to problem 14 of homework set 1. In 1000 dimensions is the answer really 5. Why 5? Can you verify that each point generated is near a corner? Is it true that at most one point will be close to any corner? If so why does one visit five corners? What if one generated points in 1000 dimensions with all coordinates 0 and 1? Should one get the same answer?